Question 1

a.

If a variable is log-normally distributed, it means that the log of the variable is normally distributed. Then, the natural level of the variable is the exponential of the normal variable, which means that it can reach a minimum of zero. It's a desirable property in some standard economic models where you assume that the gross return is log-normal distributed. The intuition is that you cannot lose more than you invest, at least for unlevered investments.

b.

Dynamic programming is a useful mathematical technique to use when you are analyzing the investor's choice of investing or consuming today, when the future is uncertain. Since the future is uncertain, the investor would not know what choices he/she would make in the future. That is of course, if the investor has a rational behavior as we in general assume. Thus, the investor can make choices regarding investing or consuming today and assume that all further choices ahead in time will be taken optimally. With the use of dynamic programming with power expected utility preferences we can derive the Euler Equation as:

$$\beta E_t(\frac{c_{t+1}}{c_t})^{-\gamma}R_{t+1} = 1.$$

What the Euler equation is saying is that the expected present marginal utility regarding two different choices is equal: Consuming one unit in this period or invest the same unit in the market and consume it in the next period instead. The market return is the relative price between the two choices.

c.

Relative risk aversion (RRA) measures how averse an investor is to risk, in relation to his/her wealth. We often assume constant RRA (CRRA), which means that the investor would be equally averse of losing a proportion of his/her fortune, even when his/her fortune are increasing or decreasing. If RRA is positive the investor will prefer a safe outcome over an uncertain outcome, when the expected outcomes are equal.

The elasticity of intertemporal substitution (EIS) can be interpreted as the investor's aversion to predictable/expected changes in consumption over time. If the agent has a high elasticity of intertemporal substitution and the expected rate of return increases, the agent will respond by investing more today and thus decrease consumption today, so that he can consume more at a later point in time.

When we use CRRA preferences RRA and EIS is dependent on each other because EIS is the inverse of RRA. So, when RRA is low, EIS is high and vice versa. That is the downside of CRRA because it doesn't distinguish between the investor's aversion to expected and unexpected changes. With Epstein-Zin (E-Z) preferences we can however distinguish between the two.

E-Z preferences are defined in term of a value function that is specified recursively, meaning that its value today depends on its value for the next period. Next, I'll show why we can distinguish between risk aversion and aversion to predictable changes when we use E-Z preferences. We start with the value function as mentioned:

$$V_t(A_t) = \max_{c_t, w_t}\left\{(1-\beta){c_t}^{1-\delta} + \beta[E_t V_{t+1}(A_{t+1})^{1-\gamma}]^{(1-\delta)/(1-\gamma)}\right\}^{1/(1-\delta)}$$

Where the recursive term $E_t V_{t+1}(A_{t+1})$ can be expressed as a function with a bad (V_B) and a good (V_G) outcome weighted with their respective probability that is 1 – q and q. Then we raise the expression to the power of 1/(1 – γ). We then obtain the certainty equivalent (CE), simply because we apply the inverse of the utility function to the expected utility. The CE is here a CES function of the good and the bad outcome, with the summation across different states of the economy:

$$CE_{t+1} = \left[(1-q)V_B^{1-\gamma} + qV_G^{1-\gamma}\right]^{1/(1-\gamma)}$$

Thus, γ can be interpreted as the relative rate of risk aversion. When we put CE back into the value function, we get a new CES function, but now with two non-stochastic variables:

$$V_t(A_t) = \max_{c_t, w_t}\left\{(1-\beta)c_t^{1-\delta} + \beta CE_{t+1}^{1-\delta}\right\}^{1/(1-\delta)},$$

that is consumption today and future consumption. Thus, δ can be interpreted as the aversion to predictable changes. It's worth noting that the inverse of δ can be interpreted as the elasticity of intertemporal substitution. The intuition behind the different type of aversions is that if δ < γ the agent has a higher tolerance for known than unknown risks, and vice versa in the opposite case.

d.

Certainty equivalence (CE) is simply the amount an investor would take with certainty, that would give the same utility as the expected utility of an uncertain choice. The investor would be indifferent between CE and the uncertain choice.

e.

Collateral plays the role as security for the lender in credit markets, but on the same time it can also play the role as input in production for the debtor. The collateral is typical some real asset as for example property, machines or cars. If the debtor defaults on his obligation, the creditor will have the right to realize the value of the collateral and hopefully get back the amount that he lended out including interests. The existing of collateral in credit markets is said to amplify the effect of business cycles. Assume that we get a negative shock that results in decreasing prices in the economy. The collateral would be worth less, which result in less credit to the debtor, which again would result in less output. Less output would again decrease the prices of the collateral even more. Thus, it's an amplifying spiral.

Question 2

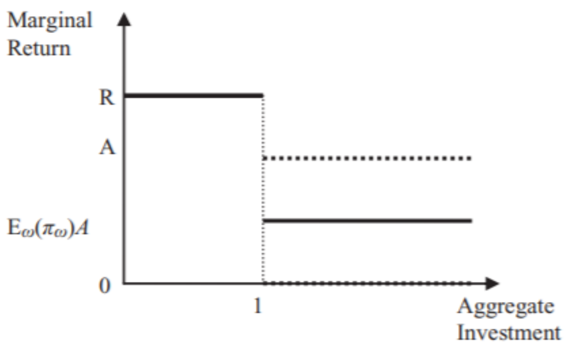
a.

Gennaiola, Shleifer and Vishny (GSV) presents a model that consist of two agents; households and intermediaries. The households represent not only households, but also pension funds, insurance firms etc. The intermediaries represent banks, shadow banks and other financial intermediaries.

GSV assume that households are infinitely risk averse (they want to maximize return in the worst-case scenario) so that they only invest in completely risk-free assets. GSV also assumes that households are a homogenic group and that they invest their money, w, by "depositing" with the intermediaries for a promised safe gross return, r. Deposits are notated as D.

The intermediaries are assumed to be risk neutral (they want to maximize expected return), their motivation is to offer a large amount of safe assets. They are professional investors that hold a given amount of equity, $\mathrm{w}_{\mathrm{int}}$, at the start of the period.

Further, GSV assume that there are two kind of investment opportunities for the households; high-quality projects ($\mathrm{I_H}$) and low-quality projects ($\mathrm{I_L}$). The high-quality projects are risk-free and gives a safe return, $\mathrm{R} \geq \mathrm{r}$, and are available in a fixed supply of 1, so that $\mathrm{I_H} \leq 1$. The low-quality projects on the other hand, are subject to idiosyncratic and aggregate risk and gives either a return, A , with a probability π, that is dependent on the state of the economy (aggregate risk), or they give a return of zero (idiosyncratic risk), with probability 1 - π. GSV assume that the economy can be in three states, with the following probability order: $\pi_{\mathrm{good}} > \pi_{\mathrm{downturn}} > \pi_{\mathrm{recession}}$. The probability π can be seen as the share of successful firms, which is higher in good times. The figure below shows the investment opportunities, where $\mathrm{E_w}(\pi_\mathrm{w})\mathrm{A}$ is the expected return of a low-quality asset that is diversified.



The intermediaries net expected cash flow is subject to funding, gross return on high-quality projects, expected gross rate on low-quality projects and the repayments that are promised to the depositors, plus the payments required to invest in high- and low-quality projects. They want to maximize the net expected cash flow given three constraints which is the core of the model; funding must cover investments and net purchases such that (T=purchase, S=sales, p=market price):

$I_H + I_L + p_H(T_H - S_H) + p_L(T_L - S_L) \leq D + w_{int}$ (1)

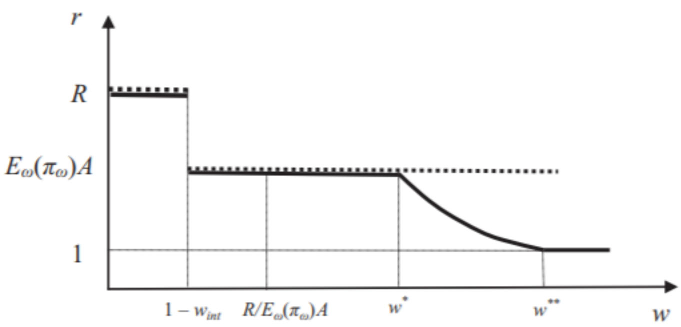
They must be able to repay deposits even in the worst-case scenario:

$R(I_H + T_H - S_H) + \pi_r A T_L \geq rD$ (2)

And they cannot securitize more than they have invested:

$S_H \leq I_H$ and $S_L \leq I_L$ (3)

In equilibrium households are promised a gross return higher than 1 and the price of securitized high-quality assets must satisfy $\frac{R}{p_H} = r$. This is an arbitrage condition that must hold given rational behavior. The price of securitized low-quality projects must be high enough such that their return will cover the payments of the promised return even in the worst-case scenario. If the demand for deposits are low, intermediaries will supply only high-quality projects and because there is no purpose of securitize these, there will be no securitization. As demand for depots increase, intermediaries will securitize low-quality projects to get rid of the idiosyncratic risk to meet the demand, and securitization will increase until there is full securitization.



The figure above shows that when demand for deposits are below $1 - w_{int}$, intermediaries will cover the promised rate of return, r, with only high-quality investments and there is therefore no need for securitization. Thus, $S_L = T_L = 0$.

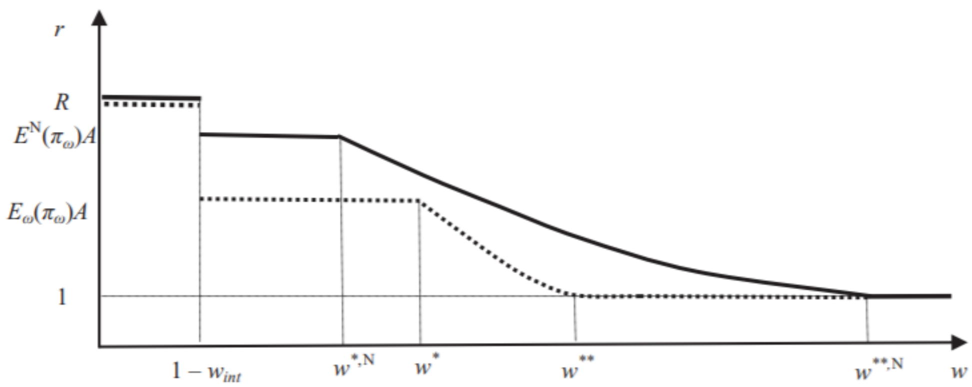
When demand for deposits are higher than $1 - w_{int}$, intermediaries will buy the maximum possible amount of high-quality projects and low-quality projects equal to $w + w_{int} - 1$. The promised return, r, is now equal to the expected return of the marginal project, which now is low-quality project. If demand for deposits are lower than $R/E_w(\pi_w)A$, the return R from high-quality projects ensures that they still meet all their obligations without securitizing. Thus, $S_L = T_L = 0$.

When demand for deposits increases further, the return R from high-quality projects will no longer cover all their obligations. Thus, the intermediaries will need to securitize low-quality projects in order to pay the promised return. Thus, $S_L = T_L > 0$. As long as the funding constraint (1) is binding, the promised return, r, will still equal the expected return of the low-quality project. The funding constraint (1) holds until $w = w^*$, where we have full securitization. Now, $S_L = T_L = I_L$. We see from the funding constraint (1) that a further increase in demand for deposits mean that the promised return, r, must decrease. When

$w = w^{**}$, the demand for deposits can no longer be supplied by the intermediaries and we have that $r = 1$.

To get a grasp of how securitization affects the economic system, GSV look at the payoff that the agents gets depending on the state of the economy. If all risk is accounted for, households will get the promised gross return. Successful intermediaries will get the return on high-quality projects, plus the return on low-quality projects, plus the return on securitized projects, subtracted the cost of funding. Thus, successful intermediaries will make profit in all states of the economy if there's not full securitization. If there's full securitization they will make profit in the good and downturn states but break even in recession. Unsuccessful intermediaries on the other hand will independent of level of securitization make profit in the good and downturn states and break even in recession. We see that if aggregate risk is fully accounted for, securitization will be a benefit for the economy as risk is allocated in a better way than before securitization. Every intermediary gets $(\pi_w - \pi_r)AS_L$ whether they are successful or not. The problems arise when tail risk is neglected.

When tail risk is not accounted for, all agents will assume that the probability of recession is zero. Intermediaries will then be overoptimistic regarding the expected return on low-quality projects



As described in the figure above, the new promised return will hence be higher or equal than it was when tail risk was accounted for, due to the now overoptimistic estimate of the low-quality projects. This causes securitization to start earlier, as the return R from high-quality projects cover less demand for deposits than before. As π_d takes the place of π_r in the funding constraint (1), intermediaries can also leverage their investments more with securitization.

Households will still get their promised gross return, but only if all intermediaries stay solvent. GSV shows in their paper that that is no longer guaranteed. Successful intermediaries will now make profit in the good and downturn states but will go bankrupt if securitization has gone too far and the economy enters the neglected recession state. For the unsuccessful intermediaries' things go even worse as they only make profit in the good state, break even in the downturn state and go bankrupt in the neglected recession state. Thus, if all agents neglect tail risk and the tail event occurs, the unsuccessful intermediaries will go bankrupt, and even the successful intermediaries will go bankrupt if securitization has gone too far. That is the downside of securitization, as it makes the financial system more fragile than before. As long as the agents are aware of all risks and judge the probabilities of bad events correct, securitization provide great benefits to the economy. However, when there's judgement errors in the probability of bad events occurring, securitization can lead to collapse of the financial system.

The figures are taken from Gennaioli, Shleifer and Vishny's paper *a model of shadow banking,* which they published in 2013.

b.

As mentioned in the presentation of the model, the microeconomic advantage of securitization is that risk is allocated through the market to the ones that are most willingly and capable of carrying it. Thus, as securitization increases intermediaries will decrease their idiosyncratic risk by buying and selling low-quality investments and eventually, they all will be exposed only to aggregate risk. That is, they will be exposed to the same risks. While this is a microeconomic advantage, it is at the same time a macroeconomic disadvantage. As outlined in the model, the disadvantage lies in the increased systemic risk that securitization provides. If aggregate tail risk is neglected, the result can be dramatic in a macroeconomic perspective, as we saw with the Global Financial Crisis. The consequence of neglected tail risk is overoptimistic expectations regarding future returns. Thus, if the tail event occurs there will be a high probability of the intermediaries going bankrupt, and as outlined in the model, they all go bankrupt if securitization has gone too far, because securitization makes them all connected, as they share the same risks. In other words, securitization is a great tool for risk allocation, but it comes with the expensive of an increased systemic risk.

c.

As I mentioned in both question a and b, the possibility of neglected aggregate risk is what makes securitization dangerous for the financial system. In the real-world tail risk doesn't happen very often, and thus it can be easy to neglect it. Especially when it's a common fact among agents that the tail risk can't happen. As we saw in the Global Financial Crisis, the risk of a collapse in the housing market was neglected by most agents as it was a common fact that house prices only could go up. So, when the collapse was a fact, most intermediaries was at the point of bankruptcy as securitization had made them all exposed to the same neglected risk. The Corona-crisis can be an example of another neglected risk as I doubt most agents had taken into account that such an event could happen. We have seen an increase in bankruptcy among smaller firms, but the financial market has seemed to recover just fine. The reason could be that securitization was not at the same level as it was under the Global Financial Crisis. Thus, even though tail risk might have been neglected, the consequence for the system as a whole was not the same due to possible less securitization.

Question 3

a and b.

The equity premium puzzle is the puzzle that the observed equity premium is much higher than what standard models predict given a reasonable level of risk aversion and risk defined as the variance in consumption. The Lucas Tree model is a simple and intuitive approach to derive the equity premium as the product of risk aversion and risk. It's intuitive because we see that either risk aversion or risk must be of a large magnitude for the equity premium to be large.

$$lnE_tR_{e,t+1} - lnR_{f,t+1} = \gamma\sigma^2,$$

The observed differences are quantitative not qualitative, meaning that the magnitude between observed and predicted equity premium differs, not that the level of risk aversion and risk does not have a contribution to the equity premium. Therefore, the theory is right, but the magnitude of the equity premium remains a puzzle. There have been many suggestions to why the observed equity premium is larger than what standard models predict. Barro's rare disaster model and Campbell and Cochrane's habit formation models are two of the suggestions.

First, I will present the two models. Second, I will discuss and compare the two models attempt to solve the equity premium puzzle.

Barros' rare disaster model

Barro's model is based upon the argument that researchers fail to consider rare but dramatic disasters. Although equity prices often are described by small fluctuations, there can be sudden drops of large scale during crisis's as we have seen through the history and just recently with the Global Financial Crisis and the Corona-Crisis. Barro argues further that investors take those events into account when they are investing in equity, while researchers typically exclude such events from their data because of their atypical behavior.

Barro's model can be analyzed within the Lucas Tree model and expected power utility preferences, but with two new elements that is the probability of a large disaster happening and the size of the disaster. Barro defines the latter as a stochastic variable because he assumes that disasters are unpredictable.

Barro shows that the equity premium consists of the product of risk aversion and risk, plus another term that is dependent on the probability of disasters:

$$\mathrm{r_e} - \mathrm{r_f} = \gamma\sigma^2 + \mathrm{pEb}[(1-\mathrm{b})^{-\gamma} - 1]$$

If the probability of a disaster happening is equal to zero, Barro's equity premium will be equal to the equity premium in the Lucas Tree model. But as the probability of a disaster happing increases, the equity premium will also increase. The last term in Barro's equation for the equity premium depends positively on the expectations of disaster size multiplied with the excess marginal utility of consumption in a disaster state over a normal state. In other words, the more discomfort that a disaster cause, the higher the equity premium will be.

Campbell and Cockranes habit formation model

As standard models look at the investors' utility level regarding current and future consumption, habit formation models look at how these levels compare to what the investor has gotten used to. This is done by replacing the variable c_t with $c_t - h_t$, where c_t is consumption at time t and h_t is the habit level at time t. The intuition is that an increase in consumption only give excess utility if the habit level does not increase with the same amount or more.

Campbell and Cochrane (C&C) use the formulation above in their contribution to solve the equity premium puzzle. Their model can be analyzed within the Lucas tree model with the help of the equity premium as covariance formula:

$$\ln E_t R_{e,t+1} - \ln R_f = -\mathrm{cov}_t(m_{t+1}, r_{e,t+1})$$

They further assume Epstein-Zin preferences and that the gross consumption growth is lognormal. Thus, they define the value as a function of wealth as:

$$V_t(A_t) = \max_{c_t, w_t}\left\{(1-\beta)(c_t - h_t)^{1-\delta} + \beta[E_t V_{t+1}(A_{t+1})^{1-\gamma}]^{(1-\delta)/(1-\gamma)}\right\}^{1/(1-\delta)},$$

where I assume that the notations are familiar to the reader. The stochastic discount factor is then given by:

$$M_{t+1} = \beta^{\frac{1-\gamma}{1-\delta}}(c_{t+1} - h_{t+1})/(c_t - h_t)^{-\delta(1-\gamma)/(1-\delta)} R_{t+1}{}^{(\delta-\gamma)/(1-\delta)}.$$

C&C the define a new variable which they call the surplus consumption ratio:

$$S_t = (c_t - h_t)/c_t.$$

If $S_t = 0$ it means that consumption is equal to the habit level, which is the worst case. If $S_t = 1$, it means that the difference between actual consumption and the habit level is so large that habits would not matter. We can now see that:

$$\frac{c_{t+1} - h_{t+1}}{c_t - h_t} = \frac{c_{t+1}}{c_t}\frac{S_{t+1}}{S_t} = x_{t+1}\frac{S_{t+1}}{S_t}$$

Furthermore, they assume that habits adjust slowly and geometrically to changes in actual consumption. They specify this in the following expression:

$$s_{t+1} = \ln S_{t+1} = (1-\theta)\bar{s} + \theta s_t + \lambda(s_t)(\ln c_{t+1} - \ln c_t - \mu),$$

where $\lambda(s_t)$ is the sensitivity function that reflects how habits responds to changes in actual consumption. C&C assume that the sensitivity function is higher or equal than zero for all s_t and higher than zero for some s_t. I will call this assumption A. By substituting into the stochastic discount factor and taking the logs they derive the equity premium as:

$$E_t r_{e,t+1} - r_{f,t+1} = -cov_t(r_{e,t+1}, m_{t+1}) = \left[\gamma + \delta\left(\frac{1-\gamma}{1-\delta}\right)\lambda(s_t)\right]\sigma^2$$

We see that if $\lambda > 0$ and both $(1 - \gamma)$ and $(1 - \delta)$ have the same sign, the equity premium will be higher than the one derived in the Lucas Tree model. I will call this assumption B. We also see that the equity premium will vary through time because of the changes in habits reflected by $\lambda(s_t)$. However, to compare the model with actual data we must use the expectations of the equation. If assumption A holds, the expectations of $\lambda(s_t)$ will definitely be higher than zero and the equity premium in this model will be higher than the one in Lucas tree model, given that assumption B also holds.

Discussion

On one hand Barros' rare disaster model tries to solve the equity premium puzzle in terms of underestimated risk, while Campbell and Cochrane's habit formation model tries to solve the same puzzle in terms of underestimated risk aversion. The merits of the disaster model are that it doesn't rely heavily on assumptions regarding parameter values. Barro is assuming that the disaster size is unpredictable, which is a realistic assumption due to the unpredictable nature of disasters. The model is overall very intuitive as it's realistic to assume that a higher probability of a disaster happening would also lead to investors demanding a higher compensation for bearing risk. However, the model is assuming that the contribution of rare disasters to the equity premium is constant. It's disregarding a psychological factor in the sense that if it's been a long time since a disaster has occurred, investors might neglect the risk of a disaster happening, whereas if a disaster has happened just recently investors might take it more into account. A time-varying contribution of rare disasters to the equity premium could be an interesting approach. Bansal and Yaron kind of does this with their model of long-term and short-term risk, but without implicit taking rare disasters into account. I will however not go further into that.

The habit formation model by Campbell and Cockrane (C&C) is on the other hand is very reliant on the magnitude of some important parameter values. They for example assumes that actual consumption cannot go under the habit level, which is not a very realistic assumption in my opinion. I've myself experienced going beyond the habit level of consumption when I went from a fulltime job to be a student. However, my habit level soon adjusted to the student life, so the assumption of varying habit level is an assumption I can relate to. In addition, the model is also heavily dependent on how the sensitivity function is specified. They never let the sensitivity function go below zero. This property has the intuition that when actual consumption is higher than the habit level by a certain level, habits no longer matter. Another property that the specification gives is that the risk-free rate is constant over time. In my opinion C&C also have a very intuitive model in the sense that consuming a unit more doesn't always give the same marginal utility. However, the magnitude of the contribution to the equity premium is heavily dependent on how the model is specified, especially the sensitivity function.

Both models are important contributions to solve the equity premium puzzle that was "discovered" by Mehra and Prescott back in 1985. And as we have learned in this course, many people consider the habit formation model from C&C as the final verdict on the puzzle. However, as I've stated above, the model comes with questions regarding the magnitude of some important parameters. In my opinion, Barros' model of rare disasters is as intuitive and also sheds light over important factors that was not taken into account by Mehra and Prescott in their original paper.

On a side note it's clear that the equity premium is driven by human behavior. Human behavior is not as easy to model as behavior that is more physical and consistent of nature. Thus, the models of human behavior are good for showing a general picture, but not as good of showing the magnitude of the behavior, as there are so many factors to model that are likely to change over time. In that sense there is a good chance that the equity premium puzzle remains just that; a puzzle, for a long period ahead.