Question 1:

a) risk aversion

Risk aversion refers to how much risk (Price volatility) an investor can/will accept. A risk averse investor seeks safe investment, safe returns rather than risky investments with possible higher returns. He/She would rather put their money in the bank with a possibly lower, but safe interest rate, compared to in stocks with possible higher returns, but also with the risk of lower returns.

b) Elasticity of intertemporal substitution

The elasticity of intertemporal substitution can be written as 1/ γ , where γ stands for the relative risk aversion. When we have power expected utility preferences EIS equals the reciprocal of the measure of relative risk aversion, which means that that risk aversion to unpredicted and predicted risk is equal. With Epstein zin preferences we distinguish between predicted and unpredicted risk, and EIS can then be written as the reciprocal of δ, one diveded by the amount of aversion to the predicted risk.

With Epstein zin we define δ as aversion to predicted changes and γ as aversion to unpredicted changes.

If $\gamma > \delta$ the investor has a higher tolerance for known than unknown risk and has more aversion towards unknown volatility. This type of investor would prefer early resolution. An investor with these types of preferences would for example prefer to know if he/she has a high probability of getting a serious illness. If $\gamma < \delta$ the investor has a higher tolerance for unknown risk and would prefer not to know if he/she has a high probability of getting a serious illness.

c) equity premium

The equity premium is defined as the difference between the expected equity return and the risk free rate. It can be looked at as a type of compensation for investors taking on risk:

$$P = ER_r - R_f$$

The equity premium can be expressed by means of a covariance:

$$R_{e,t+1} - R_{f,t+1} = -\frac{cov(M_{t+1}, R_{p,t+1})}{E_t M_{t+1}}$$

$R_{e,t+1}$ = Rate of return on equity
$R_{f,t+1}$ = risk free rate of return
γ = rate of risk aversion
M_{t+1} is the marginal utility and can also be expressed as Beta$(c_{t+1}/c_t)^{-\gamma}$, when we have power expected utility .
Beta is the subjective consumption factor and (c_{t+1}/c_t) is the growth rate of consumption.

For the equity premium to be positive, the covariance need to be negative. It has to be like this as the investors needs to be compensated for the fact that they get to consume less when they need it the most and consume more when they need it the least.

d) equity premium puzzle

People (investors) do not seem very risk averse when they are deciding on investments, but they are still willing to pay a whole lot to avoid risk. This is what makes the Equity premium puzzle, as people are getting compensated much more for taking on risk, then what they sacrifice when taking on the risk. The puzzle arises because the equity premium observed for investors investing in risky assets, exceeds the one predicted by standard theory.

In attempts of solving the equity premium puzzle, the main answer is that either the risk or the risk aversion of the investor has been underestimated. It is a quantitative problem.

e) Dynamic programming

The dynamic programing method is method used to analyze optimal decisions when both future asset returns and investors own choices in the future is uncertain. We assume power expected utility preferences, which means we do not distinguish between risk aversion and aversion to expected/known changes in consumption over time, and that the investors choices will be made rationally in the future as well as now.
We further assume that the investor has no other funding sources for consumption than financial returns.

We start with defining the future wealth, which is equal to the wealth in time t that is not spent on consumption, but invested in a portfolio consisting of both equity and risk free assets.

$$A_{t+1} = R_{p,t+1}(A_t - C_t) = (w_t R_{t+1} + (1 - w_t) R_{f,t+1})(A_t - c_t)$$

Further, the choices the investor will make today will be defined, as the future is uncertain. The investor has to determine the level of consumption c_t along with share invested in equity, wt. To do so, the value function is defined. The value function tells that value today depends on current wealth, which can be defined as consumption with power expected utility in addition to the discounted expected future value.

As the value function has been maximized, the first derivatives of ct and wt must be zero. And by derivation of ct we find that:

$c_t^{-\gamma} = \beta E_t V'_{t+1}(A_{t+1}) R_{p,t+1}$

To find the derivation of Vt+1 wrt At we find that the only way At enters the function. Is through future wealth. Derivate the future wealth function with regards to At, and can then define:

$V'_t(A_t) = \beta E_t V'_{t+1}(A_{t+1}) R_{p,t+1}$

Observe that the right side of the two last equations are equal. The marginal value of wealth is equal to the marginal utility of consumption. This must also be the case for period t+1.
We can rewrite to find the Euler equation:

$$\beta E_t(\frac{c_{t+1}}{c_t})^{-\gamma} R_{p,t+1} = 1$$

This Euler Equation tells us that the expected present marginal utility of setting aside one unit of today's consumption to invest it in the market, and then later consume it in the next period instead, should be the same as the marginal utility of consuming it now.

The first part to the right in the equation is the stochastic discount factor and can be defined as M_{t+1}.This can be inserted into the Euler eqation and simplify it further. Then we can derive the optimal portfolio choice, by differentiating wrt to equity share, wt:

$$\beta E_t c_{t+1}^{-\gamma} \quad (R_{e,t+1} - R_{f,t+1}) = 0$$

Then devide both sides by $c^{-\gamma}$ and obtain the expectations of a product. This equals the product of the expectations plus the covariance. If we assume $R_{f,t+1}$ is a constant with no variance we can solve for the equity premium as:

$$E_t(R_{e,t+1} - R_{f,t+1}) = \frac{-cov\ (\beta(\frac{c_{t+1}}{c_t})^{-\gamma}, R_{e,t+1})}{E_t \beta(\frac{c_{t+1}}{c_t})^{-\gamma}}$$

The equity premium is proportional to the negative covariance between the marginal rate of substitution between future and present consumption, Mt and the rate of equity return. The covariance is negative so that the equity risk premium is positive and If equity return is high the investor can afford to have a higher consumption in the future, so that the marginal rate of substitution is low

Question 2:

If we assume that the corona crisis can be looked at as a rare disaster, it fits in to Barros rare disaster model, where the equity premium puzzle is explained by rare disaster being underestimated/overlooked.

Barros rare disaster model explains the equity premium puzzle with risk being underestimated. He believes that most researchers fail to consider rare but dramatic disasters. Investors will however take these events into account when deciding to make a risky investement. The problem occurs as researchers who observe investors behavior tend to exlude such data because of their atypical behavior. According to this model, investors who´s considering investing in the ongoing crisis, will take the risk following the corona crisis into account when deciding on where, and on what to invest. The researchers will not, an thus for them the risk will be a lot lower. With a lower risk a lower risk premium should follow, but the actual risk premium will appear much higher than what the researchers observe, as the actual risk of Corona should be taken into account.

In Barros model normal times will occur with a probability p-1, and disaster times with a probability p.

Growth rate in normal times: $lnx_t = lng_t \sim N(\mu, \sigma^2)$
Growth rate in disaster times: $lnx_t = lng_t \sim N(1-b), 0 < b < 1$

b is defined as the size of the disaster and is a stochastic variable. We then got the growth rate: $x_t = (1-b)g_t$

their expectations are then:

$E(X_t|normal) = Eg_t = e^{\mu+(\frac{1}{2})\sigma^2}$

$E(X_t|disaster) = (1-Eb)Eg - cov(b, g_t) =$ (1-Eb)Eg

Assume that b and g is independent so that cov = 0 and find that expected growth rate is:

$$Ex_t = (1-pEb)e^{\mu+(\frac{1}{2})\sigma^2}$$

When solving for the equity premium we find the formula given in the hint:

$$r_e - r_f = \gamma\sigma^2 + pEb[(1-b)^{-\gamma} - 1]$$

We observe that if the probability of disaster is 0, the risk premium will be equal to the risk premium in the lucas tree model. The greater the probability of disaster (p), the greater the risk premium. The equity premium depends positively on the average disaster size. The term inside the square brackets is the excess marginal utility of consumption in a disaster state over a normal state. If a disaster occurs, the worse the disaster and consequences, the larger the addition to the risk premium.

Investors considering the corona crises when deciding on investments will automatically get a higher risk premium, compared to if they didn't take it into account. As long as the disaster occurs, their risk premium will exceed the risk premium that the researchers conclude with. If the economic researchers also take the disaster into account, they will probably find the same equity premium, and the equity premium puzzle could have been solved, according to Barro.

But, Barro's assumption of investors taking rare disasters into account while economic researchers don't, have little independent support from real life. A good example from where Barro's assumption do not hold, is the global financial crisis in 07-09. Investors ignored the low probability risk in e.g mortgages which was an important reasons for why the financial crisis became as severe as it did. As we do not know if investors in real life would take the corona crisis into account, it is probably not a good way to solve the equity premium puzzle.

Question 3:

In Kyotaki and More's model, capital is used for both production and as collateral and the model describes how this double use of capital can exacerbate business cycle movements. As credit is essential in the Kyotaki and More model it needs two types of agents.

- Farmers (impatient) who wants to borrow
- And gatherers (the patient ones) who are the lenders

Farmers in the model, is firms with credit constraints in the real world, and gatherers are the creditors.
In the model, the presence of the farmers in their production is essential, as their output will fail to materialize if they run away with the borrowed money (moral hazard).
Moral hazard is the source of the credit constraint in the model, caused by asymmetric information. The farmers can walk away from the land, and no one else can produce on it (assumption), thus it is just worth the price of the land. The lender (gatherer) do not know if the farmer is going to produce on his land or just run away, and therefore, to secure him/herself he will only lend the farmer the amount that equals the land area in the next period. By doing so he will not experience a loss, if the farmer run away with the money. This is the collateral constraint, and can be written as

$$Rb_t \leq q_{t+1}k_t$$

R- the market rate of interest
b_t – amount of the loand
q_{t+1} – price of the land in the next period
k_t- The amount of land that a farmer uses this year.

This is a real world problem as well, and an important reason for lenders to demand collateral from borrowers. E.g it is common that young people use their parent as collateral when buying their first home. The bank can not know it the buyer will pay down the loan, so if they fail to repay, the parent will stand as responsible…

We assume a productivity shock that lowers the value of the farmers land. As the land is used as collateral, the net value of the credit constraint firms will go down if a shock occurs.
As they have less collateral to borrow, their demand for capital goes down Then the user costs go down and the land prices go down. This spiral will continue in the following time periods.

The model shows both a static and a dynamic multiplier effect. For the market to clear, the gatherers have to buy the land, but they only increase their demand if the user cost go down. This will be repeated for all subsequent periods.

The effect of the shock in only the period where the shock uccurs, is called the static effect and is relatively small. The dynamic effect on the other hand is intertemporal and much bigger than the static effect.

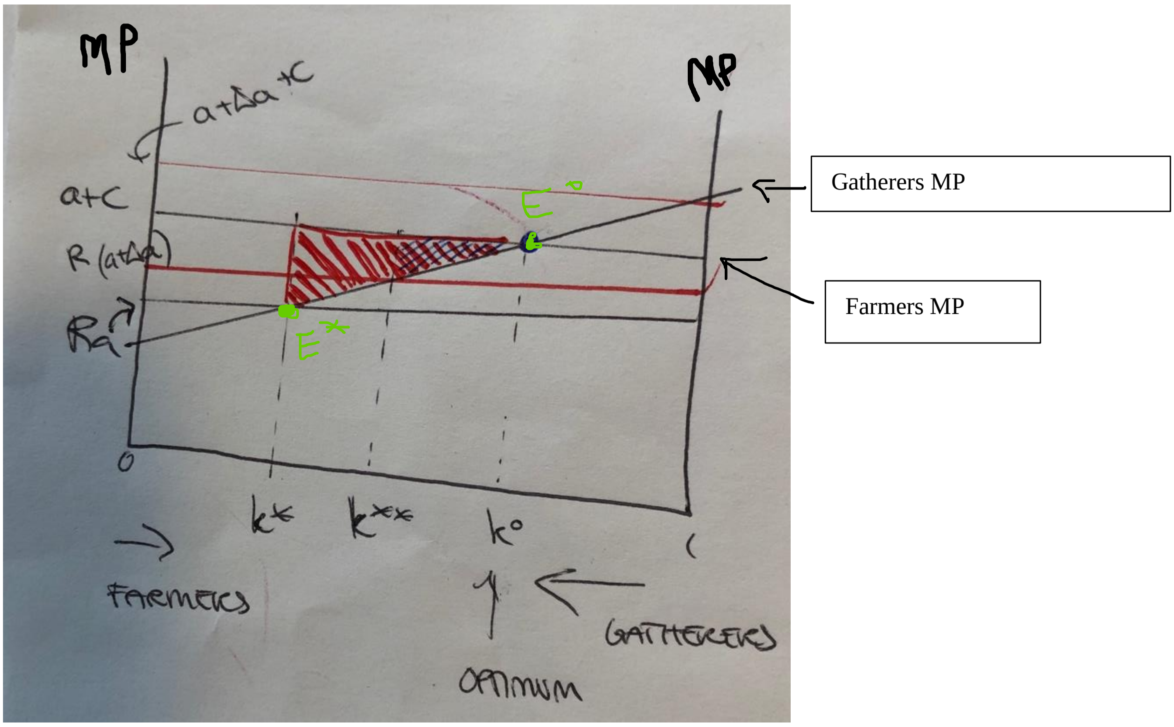
We know that The Farmers can use the land to produce more output , while the gatheres only pick the fruit that grows on their land. Land held by gatherers can be written as K+K' = 1 and land held by farmers is K.

With the collateral constraint already mentioned we get the following situation in equilibrium (steady state): E^*

- Farmers maginal productivity is bigger than the gatheres marginal productivity.
- Farmers hold less capital than what is optimal
- As optimal allocation is in E^0 We get a deadweight loss, 

because the farmers cannot obtain the credit needed to take advantage of their optimal productivity.

The deadweight loss can be erased with a government guarantee, but then the government will be left with the losses if the farmer run away. Giving subsidies to the farmers would have the same effect but could turn out very expensive as both the honest and dishonest farmers would get the payments.



We then assume that a positive shock occurs, and we go from K* to K**. The deadweight loss decreases as the farmers get an increased marginal productivity. With increased marginal productivity their net worth increases, and they can then borrow more capital. Thus, they can increase their production and again they have more capital that can be used as collateral. This effect goes on and on.
The deadweight loss is now the smaller blue/red triangle.

By looking at the change in land holding by farmers and the change of the land price, with and without the dynamic multiplier, we can observe how much the shock amplified through the credit constraint, impacts the economy.

Without credit constraint:
$\widehat{y_t^e} = \Delta$

Will have no reallocation, as both agents change by the same amount.
$\widehat{y_t^e}$ is the aggregate output

With credit constraint:

Static:
Deviation from steady state price
$\widehat{q_t} =$((R-1)/R)(1/n))* Δ

Deviation from steady state land distribution
$\widehat{K_t} = \Delta$

Dynamic:
Deviation from steady state price
$\widehat{q_t} = \frac{1}{n}\Delta$

Deviation from steady state land distribution
$\widehat{K_t} = \frac{1}{1+\left(\frac{1}{n}\right)}(1 + \left(\frac{R}{R-1}\right)\frac{1}{n})\Delta$

$$\widehat{y_t} = \Delta + (a + c - Ra)\frac{K^*}{y^*}\widehat{K_t}$$

From these results we see that the dynamic effects are what really makes the credit constraint matter as an amplifier of business cycles. The same will apply to negative shocks.
We have also observed that moral hazard is what keeps the economy from reaching the optimum.