V14

#1

a) $\int x \ln(x^2)\,dx$ (bruker at $\int f g'\,dx = fg - \int f' g\,dx$)

Sett $f(x) = \ln(x^2)$ og $g'(x) = x$:

$$\int x\ln(x^2)\,dx = \frac{1}{2}x^2\ln(x^2) - \int \frac{1}{2}x^2 \cdot \frac{1}{x^2}\cdot 2x\,dx$$

$$= \frac{1}{2}x^2\ln(x^2) - \int x\,dx$$

$$= \underline{\underline{\frac{1}{2}x^2\left(\ln(x^2) - 1\right) + C}}$$

b) $\int \frac{2\ln x}{x}\,dx$

Sett $u = \ln x$, $du = \frac{1}{x}dx$:

$$\int 2u\,du = u^2 + C.$$

Sett inn for u:

$$\int \frac{2\ln x}{x}\,dx = \underline{\underline{(\ln x)^2 + C}}$$

c) $$\frac{d}{dt}\int_{t^2}^{t^3} e^{\frac{1}{2}x}\,dx = \underline{\underline{e^{\frac{1}{2}t^3}\cdot 3t^2 - e^{\frac{1}{2}t^2}\,2t}}$$

#2

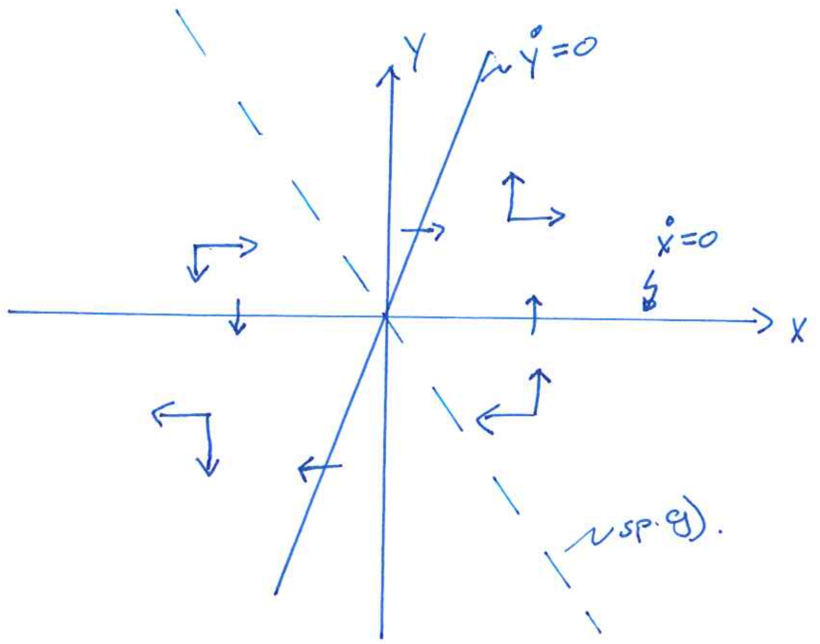
$$\dot{x} = 2y$$
$$\dot{y} = 3x - y$$

a) Vi må finne x og y slik at

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Leftrightarrow \begin{bmatrix} 2y \\ 3x-y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Leftrightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

$\Rightarrow$ Systemets likevektspunkt er $(0,0)$.

b)



c)

$$\bar{A} = \begin{bmatrix} 0 & 2 \\ 3 & -1 \end{bmatrix}$$

d)

$$|\bar{A}| = \begin{vmatrix} 0 & 2 \\ 3 & -1 \end{vmatrix} = 0\cdot(-1) - 3\cdot 2 = \underline{\underline{-6 < 0}}$$

Da er, siden $|\bar{A}| < 0$, likevektspunktet et <u>saddelpunkt</u>

e) Vi har at

$$\bar{A}\bar{x} = \lambda\bar{x} \Leftrightarrow (\bar{A} - \lambda\bar{I})\bar{x} = \bar{0} \quad (*)$$

Vi finner $P(\lambda)$ ved å beregne $|\bar{A} - \lambda I|$:

$$\begin{vmatrix} -\lambda & 2 \\ 3 & -1-\lambda \end{vmatrix} = -\lambda(-1-\lambda) - 3\cdot 2 = \lambda^2 + \lambda - 6 = P(\lambda).$$

f) Finner egenverdiene ved å sette $P(\lambda)=0$:

$$\lambda^2 + \lambda - 6 = 0$$

$$\lambda_{1,2} = \frac{-1 \pm \sqrt{1^2 - 4(1)(-6)}}{2\cdot 1} = \frac{-1 \pm 5}{2} = \underline{\underline{-3;\ 2}}$$

Setter inn for $\lambda = -3$ i $(*)$ for å finne egenvektoren tilhørende $\lambda = -3$:

$$+3x + 2y = 0$$
$$3x + 2y = 0$$

$$\left[\begin{array}{cc|c} 3 & 2 & 0 \\ 3 & 2 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 3 & 2 & 0 \\ 0 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & \frac{2}{3} & 0 \\ 0 & 0 & 0 \end{array}\right]$$

Sett $y = s$:

$$x + \frac{2}{3}s = 0 \Leftrightarrow x = -\frac{2}{3}s$$

Vi får da egenvektoren

$$\underline{\underline{\bar{x} = s\begin{bmatrix} -\frac{2}{3} \\ 1 \end{bmatrix}}}$$

g) Linjen er parallell med vektoren $\left(-\frac{2}{3}, 1\right)$.

En slik linje, som også går gjennom origo, er

$$y = -\frac{3}{2}x.$$

#3 Løs optimeringsproblemet

$$\text{maks } \left(\frac{1}{2}x - y\right)$$
$$\text{s.t} \quad x + e^{-x} = y.$$

Vi får Lagrangefunksjonen

$$\mathcal{L} = \frac{1}{2}x - y - \lambda(x + e^{-x} - y).$$

FOC:

$$\mathcal{L}_x = \frac{1}{2} - \lambda(1 - e^{-x}) = 0$$
$$\mathcal{L}_y = -1 + \lambda = 0 \Leftrightarrow \lambda = 1$$
$$\mathcal{L}_\lambda = -(x + e^{-x} - y) = 0 \Leftrightarrow y = x + e^{-x}$$

Sett inn for λ i $\mathcal{L}_x$:

$$\frac{1}{2} - 1 + e^{-x} = 0 \Leftrightarrow e^{-x} = \frac{1}{2} \Leftrightarrow -x = \ln\left(\frac{1}{2}\right) = \ln 1 - \ln 2 \Leftrightarrow$$
$$\underline{x = \ln 2}.$$

Innsatt for x i $\mathcal{L}_\lambda$:

$$y = \ln 2 + e^{-\ln 2} = \ln 2 + e^{\ln \frac{1}{2}} = \underline{\ln 2 + \frac{1}{2}}$$

Den borderte Hessematrisen er

$$B = \begin{bmatrix} 0 & -1 + e^{-x} & 1 \\ -1 + e^{-x} & -\lambda e^{-x} & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad B(\bar{x}^*) = \begin{bmatrix} 0 & -\frac{1}{2} & 1 \\ -\frac{1}{2} & -\frac{1}{2} & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Beregner determinanten

$$B_2(\bar{x}^*) = \begin{vmatrix} 0 & -\frac{1}{2} & 1 \\ -\frac{1}{2} & -\frac{1}{2} & 0 \\ 1 & 0 & 0 \end{vmatrix} = (-1)^{3+1} \cdot 1 \cdot \begin{vmatrix} -1/2 & 1 \\ -1/2 & 0 \end{vmatrix} = \frac{1}{2}$$

Vi har at $(-1)^2 \cdot B_2(\bar{x}^*) = \frac{1}{2} > 0 \Rightarrow \bar{x}^*$ løser maksproblemet.

$$\Rightarrow \underline{\underline{\bar{x}^* = \left(\ln 2, \ln 2 + \frac{1}{2}\right)}}$$

#4

$$PQ - QP = P$$

$$P^2Q - QP^2 = 2P^2$$

Vi får at

$$PQ = QP + P \longleftrightarrow QP+P$$

$$PPQ = PQP + PP \Longleftrightarrow P^2Q = \overbrace{(PQ)}^{QP+P} P + P^2$$

$$= (QP+P)P + P^2 = QP^2 + 2P^2 \Longleftrightarrow P^2Q - QP^2 = 2P^2.$$