

## Assessment guidelines -Exam SØK 2012. H19.

The grade is based on overall assessment of the answers on the exam questions

### Question 1.

- a) Opportunity costs and sunk costs are explained in textbook, ch.3.2 and 3.3, which also contain examples and explanations of the role of these concepts in rational decision-making.
- b) Loss aversion is the phenomenon that people dislike losses more than they like gains, i.e. losses loom larger than gains. This can be illustrated by value functions,  $v=v(x)$  where  $v$  represents the subjective value for the individual where  $x$  can be gains ( $x>0$ ) or losses ( $x<0$ ), for example gains or losses of income relative to some reference level.

Suppose the value function have the simple form  $v=hx$ , where  $h>0$  is a parameter

If  $h$  is larger if  $x$  is loss ( $x<0$ ) than if it is a gain ( $x>0$ ), then we have loss aversion. If  $h$  is **independent** of whether  $x$  is loss ( $x<0$ ) or a gain ( $x>0$ ), the value function does not exhibit loss aversion. Figures could be drawn to illustrate the cases.

### Question 2.

- a) The journalist makes the base-rate neglect fallacy, i.e. he does not account for the share of dopers in the population of athletes.
- b) Let  $P$  denote the event that the doping test is positive, and  $B$  the probability that the athlete is doped. Then, the probability that the athlete is doped, given that the test is positive is  $P(B|P)$ . Apply Bayes rule:

$$P(B|P) = \frac{0.95 \times 0.15}{0.95 \times 0.15 + 0.05 \times 0.85} = 0.77$$

From the formula it is easily seen that the journalist does not take into account that the dopers represents a relatively small proportion of the population of athletes when making his judgement in a).

- c) Simplest way to illustrate the law of small numbers is to use an example of a purely random event. Belief in the law of small numbers implies that subjects believe in a systematic pattern in events, even if no such patterns exist. Suppose that the return from a mutual fund is purely random with probability of Up equal to 0.5 and the probability of Down equal to 0.5, i.e. in the **long run** we expect the share of Up returns to be 0.5. Suppose we have observed a pattern

UpUpUp. A rational agent would judge the probability of a Down in the next observation to be 0.5 and base his investments decisions on that. A believer in the law of small numbers, in contrast, may falsely believe that the probability of the next observation to be less than 0.5 since three Up's are already observed and base his decisions on this (false) judgement. Other similar examples can be constructed and used, see p.100-101 in textbook.

### Question 3.

a)  $EU = pr(s_1) \times u(y_1) + pr(s_2) \times u(y_2)$ , with  $pr(s_1) + pr(s_2) = 1$  and  $u(y_i)$  representing the utility of wealth, with  $u'' > 0$ .

The candidates should be able to connect attitude towards risk to the shape of the utility function. Concave utility function  $u'' < 0$  in wealth implies risk aversion. Convexity implies risk loving ( $u'' > 0$ ) and risk neutrality implies that  $u'' = 0$ . Put another way: You are risk averse if you reject a lottery in favor of a sure amount equal to the expected value of the lottery. If you accept the lottery in favor of the sure amount, you are risk lover (risk prone). If you are indifferent between the two, you are risk neutral. Diagrams would help to illustrate this. May also use the concept of certainty equivalent to explain attitude towards risk as on p.147 in textbook.

b) This part of the question does not count for grading since there was an error in the exam text. See also information on the course site in **blackboard**

### Question 4.

Start with an intertemporal utility function defined over three periods: Let before semester start denote period 0, the semester be period 1 and the exam be period 2. Let  $U$  denote the discounted utility in period 0.

$$U = u_0 + \beta\delta u_1 + \beta\delta^2 u_2 \text{ with } u \text{ representing the utility in each period}$$

$\delta$  represents the discount factor in the standard model of intertemporal choice, while  $\beta$ , where  $0 < \beta \leq 1$  represents the degree of present biased preferences or lack of self control.

To simplify the exposition: Let  $c$  represent the cost associated with the effort exerted during semester, (period 1), while  $b$  represents the benefit received by passing exam in period 2.

As viewed from period 0 the discounted cost is  $\beta\delta c$  and the discounted benefit is  $\beta\delta^2 b$ .

Thus, since the students plan to provide the effort required to pass exam, it must be the case that  $\beta\delta^2 b > \beta\delta c$ , i.e.  $\delta b > c$ .

Consider then period 1, the period where effort is actually exerted at cost  $c$ :

The cost of providing effort is  $c$ , while the discounted benefit is  $\beta\delta b$

Thus, the decision rule whether to provide effort is  $\beta\delta b > c$

Thus, the actual decision whether to provide effort in semester (period 1) differs from that in the planning period if  $\beta < 1$ , i.e. to the extent that the student has self-control problems. In other words, a reasonable explanation of the student's different behavior at midsemester is that student A has time-inconsistent preferences and lack self-control i.e.  $\beta < 1$ , while student B has time-consistent preferences,  $\beta = 1$  and behaves according to his plan before the semester started and exerts the necessary effort during the semester to pass exam

Question 5.

The candidates should explain why the standard game theoretic model predicts contributions equal zero (Nash equilibrium) in the one-shot game and the free-rider problem in this type of games. Thus, the best response of each participant is to contribute nothing and hope that the others contribute. The pareto optimal solution is for each participant to contribute 100 and in total this will give a level of the public good at  $4 \times 2 \times 100 = 800$  and each receive 200. However, this is not a Nash equilibrium, since each participant has incentives to deviate unilaterally from this solution.

In the experiment, the individuals provided 200, thus the participants actually contribute some amount, but not the pareto-optimal level. Thus, the actual results are not fully consistent with the pessimistic view from the standard model. Students should discuss trust and reciprocity (ch 11.3 in textbook) as reasons why participants contribute at all.

Question 6.

Pure strategy Nash equilibrium implies that the players choose one action or another with certainty. In contrast, in a mixed equilibrium, the players choose actions with some probability between zero and one..

In a mixed strategy equilibrium the players must be indifferent between the pure strategies. If not, they would choose one or the other strategy with certainty.

**The consumer (you) must be indifferent between playing Water or Wine.**

i.e., your expected payoff from Wine must equal expected payoff from Water

Let  $q$  denote the probability that the police play Control and  $(1-q)$  the probability that police play No Control. Expected payoff from Wine is then:  $q \times -3 + (1 - q) \times 1$

Expected payoff from Water is 0 with certainty, thus you are indifferent between Wine and Water if  $q \times -3 + (1 - q) \times 1 = 0$ , i.e. if  $q=1/4$ .

**Police must be indifferent between playing Control and No Control**

Let  $p$  be the probability that You (consumer) plays Wine

Police expected payoff from Control is -1 with certainty

Expected payoff from No Control is  $p \times -3 + (1 - p) \times 0$

Police is indifferent between control and no-control if  $-3p = -1$ , i.e if  $p=1/3$

Thus, the Nash equilibrium in mixed strategies is that police play control with probability 1/4 and you(consumer) play wine with probability 1/3.

b) Ultimatum game: A proposer player I is given an amount of money  $X$  which can be shared with a receiver, Player II. Let  $S$  be the share proposed by player I.  $S$  is between 0 and 1. The rules of the game is as follows: If the receiver (player II) accepts the proposed  $S$ , player I keeps  $X(1-S)$ , and player II receives  $SX$ . If the receiver rejects, the game ends and both players end up with 0. Students should show that the subgame perfect Nash-equilibrium in this sequential game is for player I to propose a negligible share to player II, i.e  $S \approx 0$ , and player II will accept it.

The dictator game is similar, the difference is that stage 2 of the game is eliminated. Thus the decision for player I is to decide whether to share the cake ( $X$ ) with the other participant,

Player II, or not and how much to give. If players are only concerned with their own utility or payoff, the optimal strategy for player I is to keep X for himself.

Typically, the receiver in ultimatum games reject offers well above zero, while proposers are observed to propose offers well above zero. Similarly, in ultimatum games, offers above zero are frequently observed. This may be explained by social preferences, altruism and fairness, as discussed in chapter 11.2 in textbook. One can argue that the dictator game experiment is more likely to identify the pure role of social preferences than the ultimatum game experiment.

Question 7.

The key is to argue in terms of value function and possibility of loss aversion. System A provides bonus and teacher incentives in the gain frame, while system B provides bonus and teacher incentives in the loss frame. Effective students would refer back to the discussion of loss aversion in Question 1 and use the value functions or graphs there to argue that system B is likely to be most efficient one to incentivize teachers.

Question 8.

This is the Beauty contest game (guessing game) discussed in chapter 11.4 in the textbook. Students should argue by iterative elimination of possible Nash equilibria (NE) that leads to zero as the NE with everyone sharing the prize. While the text implies  $p=2/3$ , using any  $p$  between 0 and 1 will do in the illustration of the NE. Some of the students would probably use  $p=0.7$  as in the textbook to illustrate the NE and that is fully acceptable. The results reported for  $p=2/3$  can be rationalized by the k-level model which should be explained in terms of belief of what others believe etc.

It turns out that for individuals of type  $k=1, 2, 3$ , the predicted optimal choices when  $p=2/3$  is

$$k = 1: 50 \times p \approx 33$$

$$k = 2: 50 \times p^2 \approx 22$$

$$k = 3: 50 \times p^3 \approx 15$$

Etc. Generally, for level  $k$ , the predicted optimal choice is  $50 \times p^k$ . Ultimately, when  $k$  approaches infinity, the Nash equilibrium occur. Thus, using the k-level approach as in chapter 11.4 in the textbook, the chosen numbers can be rationalized and behavior is consistent with the hypothesis that most participants were level 1 through level 3 types.