**Exam-solution FIN3005 - Asset Pricing** fall 2012.

a) By setting $\psi = 1/\gamma$, equation (1) collapses to

$$E_t r_{t+1} - r_{f,t+1} + \frac{1}{2}\sigma_t^2 = \gamma Cov_t\left(r_{t+1}, \Delta c_{t+1}\right).$$

The risk premium is determined by the coefficient of relative risk aversion (price of risk) times the covariance of the risky return and consumption growth (quantity of risk). Risky assets earn a premium over the risk-free rate only if its return covaries positively with consumption growth. Specifically: Bad return in bad times requires compensation if an investor is to hold the asset. Good answers contain these points and elaborates.

The Equity Premium Puzzle is the empirical result of a high risk premium and low covariance requiring large and possible unrealistic values of $\gamma$. Good answers contain elaboration, and possibly dragging in the Risk-free Rate Puzzle (accepting large $\gamma$ implies high risk-free rates not observed in the data).

b) Allowing $\psi \neq 1/\gamma$ implies that the risk premium is a weighted average of the two covariances, where the covariance between return and consumption growth is also scaled by the elasticity of intertemporal substitution since it involves a time-dimension. Could help explain EPP if second covariance high. This however implies that wealth should be volatile (linked through budget constraint), which is itself a problem knowing that consumption growth is very smooth over time.

c) Students should discuss data-issues, and other models and utility functions introduced in the course.

$$\Delta w_{t+1} = r_{p,t+1} + \ln\left(1 - \frac{C_t}{W_t}\right).$$

Since ratio constant, $\Delta c_{t+1} = \Delta w_{t+1}$, so the expression for $\Delta w_{t+1}$ can be substituted for $\Delta c_{t+1}$ in equation (1). This, together with the portfolio standard deviation being $\alpha_t \sigma_t$, produces the myopic solution

$$\alpha_t = \frac{E_t r_{t+1} - r_{f,t+1} + \frac{1}{2}\sigma_t^2}{\gamma \sigma_t}$$

Good answers derives this optimal share. Further, good answers discusse the two conditions of IID returns and log-utility required (do not need both simultaneously) for a constant consumption-wealth ratio to be justifiable. The optimal share is then the myopic solution because i) IID returns means investment opportunities constant over time (no reason to adjust portfolio) ii) Log-utility means that income and substitution effect from changes in investment opportunities cancel out (no reason to adjust portfolio).

e) Intuition in equation (4) (from the lecture notes):

  - The left hand side of equation (4) is the difference between actual consumption in period $t+1$ and what was expected last period. This can effectively be thought of as the "surprise" or innovation in consumption.
  - The right hand side of equation (4) shows that any positive (negative) innovation is either a result of higher (lower) than expected portfolio return, or a revision in expectations of future returns. The first effect gives a one-for-one change in the innovation of consumption. The latter effect is controlled by $(1-\psi)$. If $\psi < 1$, and there is a positive revision in expectations of future returns, this contributes towards increasing consumption today. (A dominating income effect). If $\psi > 1$, and there is a positive revision in expectations of future returns, this contributes towards decreasing consumption today. (A dominating substitution effect).

Subtracting $c_t$ from both sides of equation (4), rearranging to get an expression for $\Delta c_{t+1}$, and substituting into equation (1), yields

$$E_t r_{t+1} - r_{f,t+1} + \frac{1}{2}\sigma_t^2 = \left(1 - \theta + \frac{\theta}{\psi}\right) Cov_t\left(r_{t+1}, r_{p,t+1}\right) + \frac{\theta(1-\psi)}{\psi} Cov_t\left(r_{t+1}, (E_{t+1} - E_t)\sum_{j=1}^{\infty} \rho^j r_{p,t+1+j}\right),$$

since $Cov_t\left(r_{t+1}, E_t\Delta c_{t+1}\right) = Cov_t\left(r_{t+1}, E_t r_{p,t+1}\right) = 0$. Utilizing further that variations in portfolio return is now due to variations in the risk-free rate, $(E_{t+1} - E_t)\sum_{j=1}^{\infty}\rho^j r_{p,t+1+j} = (E_{t+1} - E_t)\sum_{j=1}^{\infty}\rho^j r_{f,t+1+j}$, and the portfolio standard devation derived above, this equation can be rewritten to equation (5).

f) Solving equation (5) for $\alpha_t$, and rewriting the last covariance, yields

$$\alpha_t = \frac{1}{\gamma}\frac{E_t r_{t+1} - r_{f,t+1} + \frac{1}{2}\sigma_t^2}{\sigma_t^2} + \left(1 - \frac{1}{\gamma}\right)\frac{Cov_t\left(r_{t+1}, -(E_{t+1} - E_t)\sum_{j=1}^{\infty}\rho^j r_{f,t+1+j}\right)}{\sigma_t^2}.$$

The demand for the risky asset is a weighted average of two components:

1. The first component is the myopic demand. This is the usual tradeoff between expected excess return and asset risk. This component carries a weight of $\frac{1}{\gamma}$ in the total demand for the risky asset.

2. The second component is known as the *intertemporal hedging demand.* This intertemporal hedging demand is determined by the covariance of the risky asset and *reductions* in future expected risk-free interest rates relative to the asset's risk. This component carries a weight of $1 - \frac{1}{\gamma}$.

Note that the intertemporal hedging demand is zero if i) risk-free interest rates are constant (IID returns) and ii) if the investor has log-utility ($\gamma = 1$). For a highly conservative investor (high $\gamma$), the the myopic demand carries little weight. What matters is the intertemporal hedging demand. The conservative investor will only hold the risky asset if it's return covaries positively with *declines* in the risk-free interest rate. This hedges the risk of reductions in the future risk-free interest rate.