Final Exam - FIN3005 Asset Pricing

(Lecturers: Snorre Lindset and Xunhua Su)

Fall, 2014

Make the assumptions you find necessary.

Problem 1 (25%) In this problem you can use the following relation:

$$p_t = E[m_{t+1}x_{t+1}].$$

Here p_t is the time t price of asset with value x_{t+1} at time $t+1$, m_{t+1} is the time $t+1$ value of the stochastic discount factor, and $E[\cdot]$ is the expectation operator.

a) Find a general expression for the risk-free interest rate R^f.

Suppose that changes in consumption can be described by the stochastic differential equation

$$dc_t = \mu c_t dt + \sigma c_t dz_t,$$

where μ og σ are constants, $c_0 > 0$, and z is a standard Brownian motion.

b) Given c_t, show that

$$c_{t+1} = c_t e^{\mu - \frac{1}{2}\sigma^2 + \sigma z_{t+1}}, \quad z_t = 0.$$

Let the utility from consumption be given by

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}, \quad \gamma > 1.$$

Let further

$$\beta = e^{-\delta}$$

be the subjective discount factor.

c) Find an expression for the risk-free interest rate $r^f = \ln R^f$.

Assume the following parameter values: $\delta = 0.05$, $\mu = 0.03$, $\sigma = 0.1$, and $\gamma = 2$.

d) Determine the numerical value of the risk-free interest rate.

e) Many European countries have real risk-free interest rates close to zero. Find the μ that makes $r^f = 0$.

Problem 2 (25%) You are given two returns, R^i and R^j with $E[R^i] \neq E[R^j]$. Both returns are on the portfolio frontier.

a) Give a short/intuitive explanation for why these two returns can be used to *span* or *synthesize* any frontier return.

Let ρ_{m,R^k} be the correlation between the stochastic discount factor and some return R^k.

b) Using the notation from class and the textbook, show that

$$E[R^k] = R^f - \rho_{m,R^k} \frac{\sigma_m}{E[m]} \sigma_{R^k}.$$

Let R^{mv} be a frontier return and R^k and R^l two arbitrary returns.

c) Find the portfolio weight a that is such that

$$E[R^{mv}] = aE[R^k] + (1-a)E[R^l].$$

Problem 3 (16%) Answer each question with no more than 100 words.

a) Which one of the following statements is inconsistent with the efficient market hypothesis or the rationality of investors? Why?

i. In some Asian markets, stocks with a ticker that ends with 8, 88, or 888 have a higher trading volume than comparable stocks, because the pronunciation of "8" is similar to that of "lucky" in these countries.

ii. Peter Lynch (manager of the the Magellan Fund, a mutual fund in the U.S.) was one of the best fund managers in the past century, but he also lost money in many years during his career.

b) The expected returns and variances of three stocks are as follows.
Stock A: r_A, σ_A^2
Stock B: r_B, σ_B^2
Stock C: r_C, σ_C^2
where $r_A > r_B > r_C$ and $0 < \sigma_A < \sigma_B < \sigma_C$.

Which one of the following statements is false? Why?

i. For a mean-variance optimizer, stock A is preferred to any portfolio formed using stock B and stock C.

ii. For a mean-variance optimizer investing in only one stock, stock A is preferred to stock B, while stock B is preferred to stock C.

Problem 4 (34%) Consider a rational agent in a world with two periods, 1 and 2. The agent has initial wealth W in period 1. There are two states in period 2: a "bad" state and a "good" state. The good state occurs with probability 0.5. The agent is a log-utility optimizer and her expected utility is thus $\log c_1 + \frac{1}{2}\beta\big[\log c_g + \log c_b\big]$, where

β – time preference (impatience)
c_1 – the consumption in period 1
c_g – the consumption in the good state of period 2
c_b – the consumption in the bad state of period 2.

Suppose that the agent can invest in a risky asset. For \$1 invested in the risky asset in the first period, the agent will get paid $\$R_g$ in the good state and $\$R_b$ in the bad state of the second period. Assume that $R_g > 1 > R_b$.

a) (10%) What is the optimal investment in the asset? What are the optimal consumptions? (You need write down the optimization problem and solve it.)

b) (4%) How does the optimal investment change when

i. β increases;

ii. R_g increases.

Suppose that, in addition to the asset above, the agent can also invest in a risk-free asset. For \$1 in the risk-free asset, the agent will get $\$R$ in both states in the second period. Assume that $R_g > R \geq 1 > R_b$. Answer the following questions:

c) (8%) Write down the agent's optimization problem (the objective function and budget constraints). Write down the first-order conditions of the optimization problem.

d) (6%) Let $W = 1$, $\beta = 1$, $R_g = 1.2$, $R = 1$ and $R_b = 0.9$. Find the optimal portfolio and the optimal consumptions.

e) (6%) In the above question d), if the agent cannot short sell any of the assets, find the optimal portfolio and the optimal consumptions.

Comments on candidate 10016 in FIN3005 Asset Pricing (Fall 14)

April 15, 2015

Problem 1

a) OK.

b) The candidate here needs to solve the SDE for dc in order to find an expression for c_t.

c) Here the candidate makes a (small) mistake. The expression for the SDF is

$$m_{t+1} = \beta e^{-\gamma(\mu - \frac{1}{2}\sigma^2) - \gamma z_{t+1}}.$$

The expected value of m is

$$E[m_{t+1}] = \beta e^{-\gamma(\mu - \frac{1}{2}\sigma^2) + \frac{\gamma^2}{2}\sigma^2}.$$

Some algebra then gives that

$$r^f = \ln R^f = \delta + \gamma\mu - \frac{\gamma}{2}(\gamma - 1)\sigma^2.$$

d) The right answer is 10%.

e) $\mu = -0.02$.

Problem 2

a) OK, but the candidate could have exploited the fact that any return on the portfolio frontier is perfectly correlated with the SDF. Then any linear combination of returns on the frontier must also be perfectly correlated with the SDF.

b) OK.

c) OK, but the idea was to use the expression derived in problem b). By using this expression for both $E[R^k]$ and $E[R^l]$, a more explicit expression for a can be derived:

$$a = \frac{E[R^{mv}] - R^f + \rho_{m,R^l}\frac{\sigma_m}{E[m]}\sigma_{R^l}}{\rho_{m,R^l}\frac{\sigma_m}{E[m]}\sigma_{R^l} - \rho_{m,R^k}\frac{\sigma_m}{E[m]}\sigma_{R^k}}.$$

Problem 3 The candidate answers the problems very well.

Problem 4 The candidate answers the problems very well.

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Kandidat nr./*Candidate no.* 10016

Dato/*Date:* 16.12.14 Side/*Page:* 1

Emnekode/*Subject* FIN3005

Antall ark/*Number of pages:* 16

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Problem 1a)

$p_t = E[m_{t+1} x_{t+1}] \qquad m_{t+1} = \beta \dfrac{u'(c_{t+1})}{u'(c_t)}$

~~For the risk free interest rate it costs R_f to get 1\$ in the future. Thus, the payoff $x_{t+1} = 1$ and~~

For the risk free interest rate, we have that you can pay $1 = p_t$ today and get $x_{t+1} = R_f$ in payoff at $t+1$. Inserting for this in the equation for p_t we get:

$1 = E[m_{t+1} \cdot R_f]$

Since R_f is a certain payment, i.e. there is no risk involved ~~we can~~ I remove R_f from the expectation operator

$1 = E[m_{t+1}] \cdot R_f$

Solving for R_f

$\dfrac{1}{R_f} = E[m_{t+1}]$

$$R_f = \frac{1}{E[m_{t+1}]}$$

Problem 1c)

$u(c) = \frac{c^{1-\gamma}}{1-\gamma}$ $\gamma > 1$ $\beta = e^{-\delta}$ $u'(c) = c^{-\gamma}$

$c_{t+1} = c_t e^{\mu - \frac{1}{2}\sigma^2 + \sigma z_{t+1}}$

we want to find $r^f = \ln R^f$

$E[c_t] = c_0 e^{(\mu - \frac{1}{2}\sigma^2)t + \sigma z_{t+1}}$

$E[c_{t+1}] = c_t e^{(\mu - \frac{1}{2}\sigma^2) + \sigma z_{t+1}}$

$R_f = \frac{1}{E[m_{t+1}]}$ $m_{t+1} = \beta \frac{u'(c_{t+1})}{u'(c_t)}$

$u'(c_{t+1}) = \left(c_t e^{(\mu - \frac{1}{2}\sigma^2)\cancel{t} + \sigma z_{t+1}}\right)^{-\gamma}$ $z_t = 0$

$u'(c_{t+1}) = c_t e^{-\gamma(\mu + \frac{1}{2}\sigma^2)}$

$u'(c_t) = \left(c_0 e^{(\mu - \frac{1}{2}\sigma^2)t}\right)^{-\gamma}$

$u'(c_t) = c_0 e^{-\gamma(\mu - \frac{1}{2}\sigma^2)t}$

$\beta \frac{u'(c_{t+1})}{u'(c_t)} = \frac{c_0 e^{-\gamma\cancel{(\mu + \frac{1}{2}\sigma^2)t}} e^{-\gamma(\mu + \frac{1}{2}\sigma^2)}}{c_0 e^{-\gamma\cancel{(\mu + \frac{1}{2}\sigma^2)t}}} e^{-\delta}$

$\beta \frac{u'(c_{t+1})}{u'(c_t)} = e^{-\gamma(\mu + \frac{1}{2}\sigma^2)} e^{-\delta} = e^{-\gamma(\mu + \frac{1}{2}\sigma^2) - \delta}$

Using the rule for taking the expectation of a lognormal variable:

$E\left[\frac{\beta u'(c_{t+1})}{u'(c_t)}\right] = E[m_{t+1}] = e^{-\gamma^2\mu + \frac{1}{2}\gamma^2\sigma^2 - \delta}$

$R_f = \frac{1}{E[m_{t+1}]} = \left(e^{-\gamma^2\mu + \frac{1}{2}\gamma^2\sigma^2 - \delta}\right)^{-1} = e^{\gamma^2\mu - \frac{1}{2}\gamma^2\sigma^2 + \delta}$

$\ln R_f = \ln e^{-\gamma^2\mu - \frac{1}{2}\gamma^2\sigma^2 + \delta} = \gamma^2\mu - \frac{1}{2}\gamma^2\sigma^2 + \delta$

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Kandidat nr./*Candidate no.* 10016

Dato/*Date:* 16.12.14 Side/*Page:* 3

Emnekode/*Subject* FIN3005

Antall ark/*Number of pages:* 16

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Problem 1d)

In problem 1c) I fand that r_f:

$r_f = \gamma \mu - \frac{1}{2}\gamma^2\sigma^2 + \delta$

Inserting for the numbers from the assignment:

$r_f = 2 \cdot 0.03 - \frac{1}{2} \cdot 2^2 \cdot 0.1^2 + 0.05 = 0.09.$

Using the model derived in 1c) i get $r_f = 0.09$.

Problem 1e)

We want to find μ so that $r_f = 0$

$$0 = 2\mu - \frac{1}{2} \cdot 2^2 \cdot 0.1^2 + 0.05$$
$$0 = 2\mu - 2 \cdot 0.01 + 0.05$$
$$0 = 2\mu - 0.03$$
$$2\mu = 0.03$$
$$\mu = 0.015 = 1.5\%$$

For the risk free interest rate to be equal to zero, μ has to be 0.015.

Kandidat nr./Candidate no. 10016

Dato/Date: 16.12.14 Side/Page: ~~4~~ 5

Emnekode/Subject FIN3005 Antall ark/Number of pages: 16

Denne kolonnen er forbeholdt sensor
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Problem 2a)

R^i and R^j $E[R^i] \neq E[R^j]$

The portfolio frontier is the hyperbola in the σ, μ axiomatic space where all efficient ~~investments~~ portfolios lie. ~~Efficient invest~~ An efficient portfolio is a portfolio for which you cannot get a higher expected return without also increasing risk or variance.

~~In this assign Assuming that both R^i and R^j are returns on portfolios made up of an arbitary numbe amount of assets in the market~~

~~If we have~~ Assuming we have complete markets and R^i and R^j are returns on ~~portfolios~~ two different portfolios that include all the assets in the market but with different weights, ~~Then any a~~ We know that all portfolios combinations of two assets will be on the frontier. We can therefore use R^i and R^j to synthesize all returns on the frontier because the frontier will be made up of ~~two~~ combinations of R^i and R^j with different portfolio weights

Problem 2b)

We want to show that

$$E[R^k] = R^f - \rho_{m,R^k} \frac{\sigma_m}{E[m]} \sigma_{R^k}$$

We know that, the basic pricing equation is given by

$P_t = E[x_{t+1} m_{t+1}]$ $m_{t+1} = m$

For the asset k, $x_{t+1} = R_k$, therefore we have that

$P_t = E[R^k m]$

The covariance identity says that

$$E[R^k m] = E[R^k]E[m] + \rho_{m,R_k}\sigma_{R_k}\sigma_m$$

$$P_t = E[R^k]E[m] + \rho_{m,R_k}\sigma_{R_k}\sigma_m \quad | : E[m]$$

$$\frac{P_t}{E[m]} = E[R^k] + \rho_{m R_k}\frac{\sigma_m}{E[m]}\sigma_{R_k}$$

$E[R^k]E[m]$ is what the asset would cost if it were risk free. Therefore $\frac{P_t}{E[m]} = R^f$

$$R^f = E[R^k] + \rho_{m,R_k}\frac{\sigma_m}{E[m]}\sigma_{R^k}$$

Thus we have that

$$E[R^k] = R^f - \rho_{m,R_k}\frac{\sigma_m}{E[m]}\sigma_{R^k}$$

This is the portfolio frontier.

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Kandidat nr./*Candidate no.* 10016

Dato/*Date:* 16.12.14 Side/*Page:* 7

Emnekode/*Subject* FIN3005 Antall ark/*Number of pages:* 16

Denne kolonnen er forbeholdt sensor

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Problem 2c)

We want to find a such that a combination for R^k and R^l will be on the portfolio frontier.

We know that

$$E[R^{mv}] = R^f - \rho_{m,R^k} \frac{\sigma_m}{E[m]} \sigma_{R^{mv}}$$

~~and that these returns will be on the frontier.~~

~~Meter~~ $\cancel{\sigma_{mv} = \sqrt{\sigma_k^2 a^2 + \sigma_l^2 (1-a)^2 + 2a^2(1-a)^2 \sigma_k \sigma_l \rho_l}}$

$\cancel{a E[R^k] + (1-a) E[R^l] = R_f - \rho_{m,R^k} \frac{\sigma_m}{}}$

~~Thus $E[R^k]$ will be given by~~

$$\cancel{E[R^k] = R^f - \rho_{mR^k} \frac{\sigma_m}{E[m]} \sigma_{R^k}}$$

$$\cancel{E[R^l] = R^f - \rho_{mR^l} \frac{\sigma_m}{E[m]} \sigma_{R^l}}$$

~~Inserting for $E[R^k]$ and $E[R^l]$ and solving for a:~~

$$\cancel{E[R^{mv}] = a\left(R_f - \rho_{mR^k} \frac{\sigma_m}{E[m]} \sigma_{R^k}\right) + (1-a)\left(R_f \; \rho_{mR^l} \frac{\sigma_m}{E[m]} \sigma_{R^l}\right)}$$

$E[R_{mv}] = a$

$$E[R^{mv}] = aE[R^k] + E[R^l] - aE[R^l]$$

$$a\left(E[R^k] - E[R^l]\right) = E[R^{mv}] + E[R^l]$$

$$a = \frac{E[R^{mv}] + E[R^l]}{E[R^k] - E[R^l]}$$

Problem 3a)

The efficient market hypothesis (EMH) states that in financial markets, prices reflect all available information so that profits can only be made from publicly available information. Statement i) is inconsistent with the EMH and rationality of investors because it implies the existence of noise traders. Buying a security because of its ~~name~~ ticker symbol, means that you are not buying it because of its fundamental value. This implies that the investors are irrational. If there are limits to arbitrage in the market, statement ii) is also inconsistent with EMH as well as rationality of investors

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Problem 3b)

Statement i) is false. The ~~covariance~~ variance of a portfolio with two stocks is given by the following formula

$$\sigma_p^2 = x_B^2 \sigma_B^2 + x_C^2 \sigma_C^2 + 2 x_B x_C \sigma_B \sigma_C \rho_{BC}$$

Thus, we can see that the risk or variance of the portfolio depends on the correlation between the two stocks. A mean-variance optimizer will choose the portfolio with the lowest variance for a given return. By forming a portfolio of stock B and C, the investor might be able to create a portfolio with $\sigma_p < \sigma_A$. Given the investor's chosen level of return and risk aversion, he may prefer ~~this portf~~ the portfolio of stock B and C with the lowest return to a portfolio of stock A.

Problem 4 a)

The investor will want to maximize expected utility given a budget constraint

$$\max_{c_1, c_g, c_b} \log c_1 + 0.5\beta \log c_g + 0.5\beta \log c_b$$

$$s.t \quad c_1 + c_g + c_b \leq w$$

$c_1 = w - x$

$c_g = xR_g$

$c_b = xR_b$

To solve this problem ~~we can~~ I insert for c_1, c_g and c_b in the objective function.

$$\max_x \log(w-x) + 0.5\beta\log(xR_g) + 0.5\beta\log(xR_b)$$

F.O.C

$$\frac{\partial}{\partial x} = -\frac{1}{w-x} + \frac{0.5\beta \cancel{R_g}}{x\cancel{R_g}} + \frac{0.5\beta \cancel{R_b}}{x\cancel{R_b}} = 0$$

Solve for x:

$$-\frac{1}{w-x} + \frac{\beta}{x} = 0$$

$$\frac{\beta}{x} = \frac{1}{w-x}$$

$$\beta = \frac{x}{w-x}$$

$$\beta w - \beta x = x$$

$$\beta w = x(1+\beta)$$

$$x = w \cdot \frac{\beta}{1+\beta}$$

To find optimal consumptions, insert for x in c_1, c_b and c_g:

$$c_1^* = w - \frac{w\beta}{1+\beta} = w\left(1 - \frac{\beta}{1+\beta}\right) = w\left(\frac{1+\beta}{1+\beta} - \frac{\beta}{1+\beta}\right) = w \cdot \frac{1}{1+\beta}$$

NTNU

Kandidat nr./*Candidate no.* 10016

Dato/*Date:* 16.12.14 Side/*Page:* 11

Emnekode/*Subject* FIN3005

Antall ark/*Number of pages:* 16

Denne kolonnen er forbeholdt sensor

This column is for external examiner

Problem 4a) continued.

$$c_g^* = x R_g = \frac{w \cdot \beta}{1+\beta} \cdot R_g = \frac{R_g \beta w}{1+\beta}$$

$$c_b^* = x R_b = \frac{R_b \beta w}{1+\beta}$$

To sum up, the optimal investment in the asset is

$$x = \frac{\beta}{1+\beta} \cdot w$$

The optimal consumptions are

$$c_1^* = w \cdot \frac{1}{1+\beta} \qquad c_g^* = \frac{\beta R_g w}{1+\beta} \qquad c_b^* = \frac{R_b \beta w}{1+\beta}$$

Investment ~~is indepent~~ in x is independent of future expected returns. Therefore this shows that with the given probabilities and utility function, the investor is myopic.

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Problem 4b)

i) β is the time preference of the investor or his impatience. We can see from the expression for x that if β increases then the investment in ~~x~~ the asset will also increase.

ii) If R_g increases, the investment in x will stay the same. Investment in x does not depend on expected return. This means that the investor is myopic.

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Problem 4c)

$R_g > R \geq 1 > R_b$

In essence, the investor is still faced with the same maximization problem as previously stated

$$\max_{c_1, c_g, c_b} \log c_1 + 0.5\beta \log c_g + 0.5\beta \log c_b$$

$$\text{s.t.} \quad c_1 + c_g + c_b \leq w$$

However, now the consumptions ~~can be~~ should be rewritten as

$c_1 = w - x - y$ where x is amount invested in risky

$c_g = R_g x + R y$ asset and y is amount invested in

$c_b = R_b x + R y$ risk free asset

Inserting for c_1, c_b and c_g in the objective function, the maximization problem then is

$$\cancel{\max_{x,y} \log(w-x-y) + 0.5\beta(R_g x + R y) + 0.5\beta(}$$

$$\max_{x,y} \log(w-x-y) + 0.5\beta \log(R_g x + R y) + 0.5\beta \log(R_b x + R y)$$

F.O.C

1) $$\frac{\partial}{\partial x} = -\frac{1}{w-x-y} + \frac{0.5\beta R_g}{R_g x + R y} + \frac{0.5\beta R_b}{R_b x + R y} = 0$$

2) $$\frac{\partial}{\partial y} = -\frac{1}{w-x-y} + \frac{0.5\beta R}{R_g x + R y} + \frac{0.5\beta R}{R_b x + R y}$$

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Problem 4d)

$w=1 \quad \beta=1 \quad R_S=1.2 \quad R=1 \quad R_b=0.9$

~~Inserting for w, β, R_S, R and R_b in F.O.C. 1)~~ and solving ~~for x~~ setting the two FOC's equal to each other

1) $\cancel{\frac{1}{1-x-y} + \frac{0.5 \cdot 1 \cdot 1.2}{1.2x+y} + \frac{0.5 \cdot 1 \cdot 0.9}{0.9x+y} = 0}$

$\cancel{-\frac{1}{1-x-y} + \frac{0.6}{1.2x+y} + \frac{0.45}{0.9x+y} = 0}$

1+2) $-\frac{1}{1-x-y} + \frac{0.6}{1.2x+y} + \frac{0.45}{0.9x+y} = -\frac{1}{1-x-y} + \frac{0.5}{1.2x+y} + \frac{0.5}{0.9x+y}$

$\frac{0.6}{1.2x+y} + \frac{0.45}{0.9x+y} = \frac{0.5}{1.2x+y} + \frac{0.5}{0.9x+y}$

$\frac{0.1}{1.2x+y} = \frac{0.05}{0.9x+y}$

$0.1(0.9x+y) = 0.05(1.2x+y)$

$0.09x + 0.1y = 0.06x + 0.05y \qquad | -0.06x - 0.1y$

$0.03x = -0.05y \qquad | : 0.03$

$x = -1.667y = -\frac{5}{3}y$

Inserting for x in F.O.C 1)

$-\frac{1}{1-(-1.667)y-\cancel{x}} + \frac{0.6}{1.2(-1.667y)+y} + \frac{0.45}{0.9(-1.667y)+y} = 0$

$-\frac{1}{1+0.667y} + \frac{0.6}{-2y+y} + \frac{0.45}{-1.5y+y} = 0$

$-\frac{1}{1+0.6667y} + \frac{0.6}{-y} + \frac{0.45}{-0.5y} = 0$

$\frac{1.5}{-y} = \frac{1}{1+0.6667y}$

$1.5(1+0.6667y) = -y$

$1.5 + y = -y \qquad | -y$

$-2y = 1.5$

$y = -0.75 \qquad x = -\frac{5}{3}(-0.75) = 1.25$

NTNU

Kandidat nr./*Candidate no.* 10016

Dato/*Date:* 16.12.14 Side/*Page:* 15

Emnekode/*Subject* FIN3005

Antall ark/*Number of pages:* 16

Denne kolonnen er forbeholdt sensor

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Problem 4d) continued.

$y = -0.75 \quad x = 1.25$

$c_1^* = 1 - 1.25 - (-0.75) = 1 - 1.25 + 0.75 = 0.5$

$c_g^* = R_s x + R y = 1.2(1.25) + 1 \cdot (-0.75) = 0.75$

$c_b^* = R_b x + R y = 0.9(1.25) + 1 \cdot (-0.75) = 0.375$

The optimal portfolio is to borrow -0.75 and invest 1.25 in the risky asset. This leads to ~~a~~

$c_1^* = 0.5$, $c_g^* = 0.75$ and $c_b^* = 0.375$

NTNU

Kandidat nr./*Candidate no.* 10016

Dato/*Date:* 16.12.14 Side/*Page:* 16 ~~15~~

Emnekode/*Subject* FIN3005

Antall ark/*Number of pages:* 16 ~~15~~ ~~15~~

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Problem 4e)

The restriction here is that $y \geq 0$

Relative risk aversion is given by

$$RRA = -c \frac{u''(c)}{u'(c)}$$

~~$u''(c)$~~

$$u'(c) = \frac{1}{c}$$

$$u''(c) = -c^{-2}$$

$$RRA = -c \cdot \left(\frac{-c^{-2}}{(c^{-1})} \right) = -c \left(-\frac{1}{c} \right) = 1$$

Given that the utility function implies constant relative risk aversion, the investor will choose ~~to~~ the investment with the highest expected return. Since the risky investment has a higher expected return than the risk free (1.05 to 1) the investor will invest everything in x

The optimal consumptions and x are therefore given by problem 1a)

$$x = \frac{1}{1+1} \cdot 1 = \frac{1}{2}$$

$$c_1^* = \frac{1}{2} \qquad c_g^* = \frac{1.2}{2} = 0.6 \qquad c_b = 0.45$$

~~However, my reasoning does not seem to make sense since c_1^* c_g^* and c_b^*~~