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Question 1

The model we have here represents the New Keynsian theory, ~~a)~~ and in this case we have a model with flexible prices

a) Profit of the firms are of the form (in general case):

$$\pi_i = \underbrace{p_i y_i}_{\text{revenue}} - \underbrace{w_i y_i}_{\text{variable cost}}$$, which in this example becomes:

$$\pi_i = p_i y \frac{p^5}{p_i^5} - w_i y \frac{p^5}{p_i^5} = y p^5 \cdot p_i^{-5}(p_i - w_i)$$

We differentiate profit with respect to price:

$$\frac{d\pi_i}{dp_i} = (y p^5 \cdot p_i^{-5})'(p_i - w_i) + (y p^5 \cdot p_i^{-5})(p_i - w_i)' =$$

$$= -5 y p^5 p_i^{-6}(p_i - w_i) + y p^5 \cdot p_i^{-5} = 0$$

$$y p^5 p_i^{-5}(-5 p_i^{-1}(p_i - w_i) + 1) = 0$$

$$\frac{5}{p_i}(p_i - w_i) = 1$$

$$5 - \frac{5 w_i}{p_i} = 1$$

$$\frac{5 w_i}{p_i} = 4$$

$$\frac{w_i}{p_i} = \frac{4}{5}$$

$p_i = \frac{5}{4} w_i$ Optimal price

$\mu = \frac{5}{4} > 1$ Markup of price over marginal cost

since $\mu > 1$, then price of demand elastisity - $\theta > 1$ and monopolistically competitive equilibrium exists.

b) <u>Labour demand:</u>

$y_i = e^{\xi} h_i$, so

$h_i = y_i e^{-\xi}$

- Variable cost: $V_i = w_i \cdot h_i = w_i \cdot y_i e^{-\xi}$
- Marginal cost: $S_i = + w_i e^{-\xi} = \frac{p_i}{\mu}$

So $\frac{w_i}{p_i} = \frac{e^{\xi}}{\mu}$ and

$\boxed{\frac{w_i}{p} = \frac{e^{\xi}}{\mu}}$ Labour demand (since all firms are symmetrical $p_i = P$)

c) <u>Labour supply</u>

$U = \ln C - h$

$\boxed{\frac{v'(h)}{u(C)} = \frac{1}{1/C} = C}$ Labour supply (is determined as relationship between marginal disutility of work and utility of consumption!

d) In equillibrium demand for labour equals labour supply:

$\frac{w_i \cancel{p}}{p} = \frac{e^{\xi}}{\mu} = \frac{v'(h)}{u(C)} = C$

$\frac{e^{\xi}}{\mu} = C$ or ~~[illegible]~~ $C = \frac{e^{\xi}}{\mu}$

Since in equillibrium: $c_i = C = y_i = Y$, then

$\boxed{Y = \frac{e^{\xi}}{\mu}}$ The equillibrium level of output

We can also write it in logarithmic form:

$\boxed{\ln Y^n = \ln e^{\xi} - \ln \mu = \xi - \ln \mu}$, where

Y^n = natural output.

e) We can see that in equillibrium with flexible prices the output <u>isn't constant</u>. It's not even a trend. The output fluctuates ~~a~~ up and down with supply ~~s~~ (and demand) shocks (ξ_u). This means that the equillibrium we have here isn't constant it's ~~changig~~ changing. If it will be a crisis, the economy will <u>recover</u> from it on its own since all firms are free to adjust prices whenever they want to market conditions.

There are two major kinds of shocks that influence the equillibrium output:

1) shock to labour supply: if people can't work more, labour supply will decline, the output will then decline. To motivate people to work wages need to be increased.

2) productivity shock: if new technology is ~~more eff~~ introduced, the production is more efficient so more can be produced with same ~~resources~~ labour.

The general model also specifies demand shocks which are shocks to consumption. Example: sudden urge to consume more among people. Than demand for that specific product rises and it'll be produced more of that product and less of others.

In general, in model with flexible prices the output gap is equal to zero (unlike the model with staggered pricing where output gap is negative) and there is no Philips curve.

f) The Taylor rule:

$$\hat{i}_t = \varphi \pi_t , \quad \varphi > 1$$

$$\hat{i}_t = E_t \pi_{t+1} + \hat{r}_t$$

$\varphi \pi_t = E_t \pi_{t+1} + \hat{r}_t$ solve for π_t:

$$\pi_t = \left(\frac{1}{\varphi}\right)\left(E_t \pi_{t+1} + \hat{r}_t\right)$$

$$\pi_t = \left(\frac{1}{\varphi}\right)\left(E_t \pi_{t+1} + \hat{r}_t\right) = \left(\frac{1}{\varphi}\right)\left(\left(\frac{1}{\varphi}\right)\left(E_t \pi_{t+2} + E_t \hat{r}_{t+1}\right) + \hat{r}_t\right) =$$

$$= \left(\frac{1}{\varphi}\right)^2 E_t \pi_{t+2} + \left(\frac{1}{\varphi}\right)^2 E_t \hat{r}_{t+1} + \left(\frac{1}{\varphi}\right) \hat{r}_t =$$

$$= \left(\frac{1}{\varphi}\right)^2 \left(\left(\frac{1}{\varphi}\right)\left(E_t \pi_{t+3} + E_t \hat{r}_{t+2}\right)\right) + \left(\frac{1}{\varphi}\right)^2 E_t \hat{r}_{t+1} + \left(\frac{1}{\varphi}\right) \hat{r}_t = \ldots\ldots =$$

$$= \left(\frac{1}{\varphi}\right)^{T-t} E_t \pi_T + \sum_{j=0}^{t-1} \left(\frac{1}{\varphi}\right)^{j+1} \left(E_t \hat{r}_{t+j} + \varphi \pi_t^*\right)$$, π_t^* = target inflation

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If Taylor's principal is valid: $\varphi > 1$, then as $T \to \infty$, $\left(\frac{1}{\varphi}\right)^{T-t} \to 0$, so we have:

~~$\pi_t = (\frac{1}{}$~~ $$\boxed{\pi_t = \sum_{j=0}^{t-1} \left(\frac{1}{\varphi}\right)^{j+1} \left(E_t \hat{r}_{t+j} + \varphi \pi_t^*\right)}$$

The way to controll inflation through monetary policcy is: If inflation rises above desired level, ~~t~~ increase nominal interest rates even more. If inflation is lower than desired level, reduce nominal interest rates even more. So it'll always be a change ~~i~~ in nominal interest rates in response to considerable changes in inflation.

The better alternative to this policy is the hybrid rule:

$$\boxed{\hat{i} = \hat{r}_t + \pi_{t+1}^* + \varphi_\pi(\pi_{t+1} - \pi_{t+1}^*)}, \ \varphi_\pi > 1$$

where π_{t+1} = actual inflation ~~this~~ next period

π_{t+1}^* = target inflation for next period.

By announcing this rule the government (Central bank) tells people that if they choose any other equilibrium than desired, it'll take action. Since agents are indifferent between ~~equil~~ ~~eq~~ equilibria, they choose the equillibrium that Central bank wants them to choose, fearing actions from Central bank. So the Central bank achieves $\pi_t = \pi_t^* \ \forall t$ without even executing the threat.

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Question 2

The New Keynsian theory represents economy without financial complicatoons and tradeoffs that accur in the real world. Among this complocations are: collateral constraint, financial multiplier effect, bank runs, financiall accelerator effect, and aggregate risk.

The models, described further, consider these real-life problems that can lead to recession of the whole economy.

① The Model of Credit cycles

In this model there're 2 goods in the economy:

1) durable asset: land (doesn't depreciate, and is in fixed supply $\bar{K}$)

2) non-durable commodity: fruit (grows on land and can't be stored)

There are 2 kinds of infinitively lived agents:

1) farmers = 1
2) gatherers = m
} both consume and produce fruit.

Both farmers and gatherers are risk-neutral with utility functions respectively:

$$E_t = \sum_{s=0}^{\infty} \beta^s x_{t+s} \quad \text{and} \quad E_t = \sum_{s=0}^{\infty} \beta'^s x'_{t+s}$$

Assumption 1: $\boxed{\beta < \beta'}$ - farmers are more impatient than gatherers and don't want to postpone production.

There are 2 markets:

1) Competitive spot market: where fruit is traded for land at a price q_t;

2) 1-period credit market: where fruit at t is traded for ~~to~~ a claim to R_t fruit at $t+1$.

In equillibrium farmers borrow from gatherers (so farmers are credit constrained and gatherers aren't credit constrained), so $R_t = \frac{1}{\beta'} = R$.

~~Th~~ Farmer and gatherer have different production technology.

Farmers production function:

$y_{t+1} = F(k_t) = (a+c)k_t$, where ak_t = tradable fruit, ck_t = non-tradable fruit,

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$\frac{a}{a+c} < \beta$ maximum savings rate for a farmer.

Assumption 2 : $c > \left(\frac{1}{\beta} - 1\right)a$

Which means that in equilibrium farmers will not consume more than the output of non-tradable fruit. All output of tradable fruit will be invested in land.

Assumptions about farmer:

1) farmer owns idiosyncratic technology, so only he can make the land produce fruit.

2) farmer is free to withdraw his labour whenever he wants.

If farmer is excessively levered, all his fruit will go to gatherers to repay the debt. Farmer therefore has incentive to withdraw his labour. Creditors (gatherers) will collaterize the land, but ~~it~~ it's worth less without farmers labour, So ~~farmer~~ farmer can bribe gatherers to keep his land and ~~beg~~ renegotiate his loan down to liquidation value. Creditors know about this, so they'll never lend more than farmers land will be worth next period without fruit:

$\boxed{Rb_t \leq q_{t+1} k_t}$ Borrowing constraint.

Farmers flow-of-funds constraint:

$$\underbrace{q_t(k_t - k_{t-1})}_{\text{purchase of new land}} + \underbrace{Rb_{t-1}}_{\text{repay old debt}} + \underbrace{x_t - ck_{t-1}}_{\text{net consumption needs}} = \underbrace{ak_{t-1} + b_t}_{\text{tradable fruit + new debt}},$$

The gatherers are symmetrical and have a production function: $y_{t+1} = G(k_t')$, $G' > 0$, $G'' < 0$.

Gatherers budget constraint:

$q_t(k_t - k_{t-1}) + Rb_{t-1} + x_t = G(k_t') + b_t$, where

$\left.\begin{matrix} b_{t-1} < 0 \\ b_t < 0 \end{matrix}\right\}$ since gatherers are lenders to farmers, In equillibrium farmer will invest all his tradable output in land and consume only non-tradable output:

$$x_t = ck_t$$

$$Rb_t = q_{t+1} \cdot k_t$$

So land demand of farmers:

$$k_t = \underbrace{\frac{1}{q_t - \frac{1}{R}q_{t+1}}}_{=u_t,\ \text{down payment per unit of land.}} \underbrace{\left[(a+q_t)k_t - Rb_t\right]}_{\text{net worth}}$$

Aggregate demand for land and borrowing for farmers:

$$K_t^* = \frac{1}{u_t}\left[(a+q_t)K_t - RB_t\right]$$

$$B_t = \frac{1}{R}q_{t+1} \cdot K_t$$

Gatherers demand for land is determined where marginal gain from land equals user cost:

$$\frac{1}{R}G(\hat{k}_t)\, u_t,$$

The equillibrium condition:

$$q_t - \frac{1}{R}q_{t+1} = u(K_t),\quad u(K_t) = \frac{1}{R}G'\left(\frac{1}{m}(\bar{K} - K_t)\right).$$

In equillibrium we have

1) $\frac{R-1}{R}q^* = u^* = a$

2) $\frac{1}{R}G'\left(\frac{1}{m}(\bar{K} - K_t^*)\right) = u^*$

3) $B^* = \frac{a}{R-1}K^*$

Since down payment per unit of land equals tradable output, in equillibrium farms neither expand or shrink.

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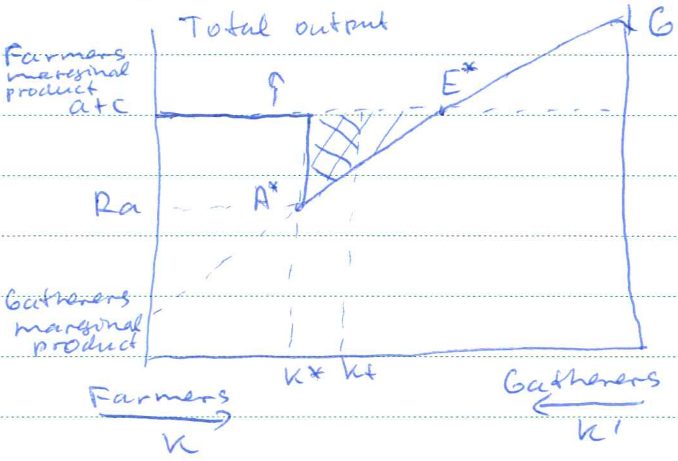
Graphic representation of equilibrium



We can see that with collateral constraint total output ≠ optimal output

▨ is loss due to collateral constraint

▦ · By how much output can be improved if farmers get more land.

In absence of collateral constraint the optimal level would be at E*, but with collateral constraint the equillibrium output is in A*.

If farmers demand for land increases due to a positive productivity shock, then the total output will grow by ▦.

Let's consider a positive productivity shock, Δ:

Combining farmers borrowing constraint and demand for land;

$u(k_t|k_t = (a + \Delta a + q_t - q^*)k^*$, which transforms into:

$$\left(1+\frac{1}{\eta}\right)\hat{k}_t = \Delta + \underbrace{\boxed{\frac{R}{R-1}}}_{\text{leverage effect.}}\hat{q}_t$$

$\left(1+\frac{1}{\eta}\right)\hat{k}_{t+s} = \hat{k}_{t+s-1}$ so effects of shocks persist into the future.

- How does land price and land usage change after shock:

If we ~~only~~ have dynamic/intertemporal multiplier:

$\hat{q}_t = \frac{1}{\eta}\Delta$ Price change is proportional to shock

$$\hat{k}_t = \left(\frac{1}{1+\frac{1}{\eta}}\right)\left(1 + \overset{\text{leverage effect: large}}{\boxed{\frac{R}{R-1}}}\cdot\frac{1}{\eta}\right)\Delta$$ change in land usage is large because of leverage effect and shock size,

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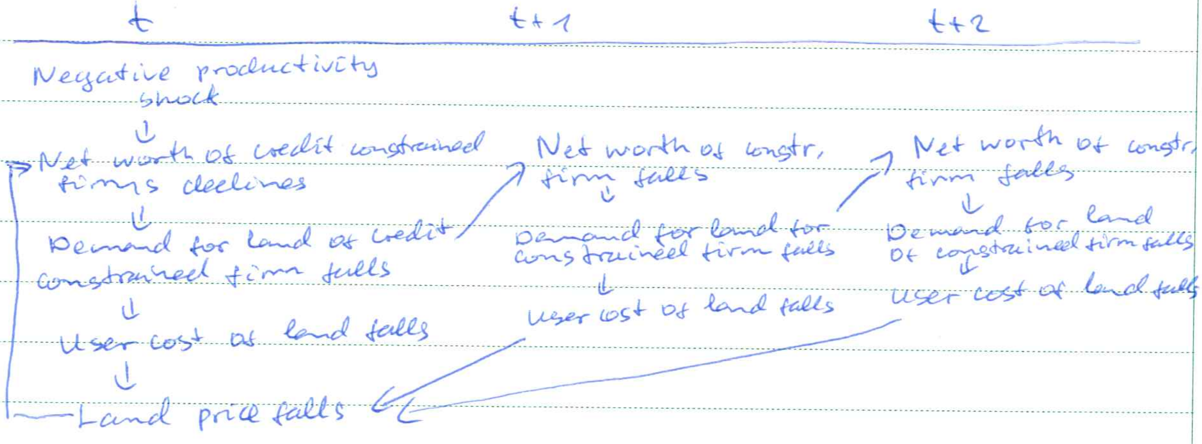
In case without collateral constraint we would only have static / ~~pa~~ within-period multiplier :

$\hat{q}_t = \frac{R-1}{R} \cdot \frac{1}{\eta} \Delta$ Price change is much smaller

$\hat{k}_t = \Delta$ Change in land usage is much smaller.

So the model of credit cycles introduces collateral constraints, This constraint makes firms that are credit constraint unable to borrow more in crisis times, It also ~~gives~~ is the ~~a~~ reason that we get dynamic multiplier, which enhances effect of shocks,

So if the form runs into a crisis being credit constrained, it can't borrow more and has to cut down on investments which leads to less revenues next period and firms net worth falls even further, To demonstrate how dynamic and static multiplier work together :



So we can see that due to dynamic multiplier a temporary small productivity shock influences demand for land of constrained firm not only in t, but t+1, t+2 .... Land price in current period gets dependent on user cost of land in next periods and that all influences the current net worth of the firm. In case of recession this firm has therefore a much bigger chance to default than it would be ~~if not for ca~~ without credit constraints

2) $\boxed{\text{Banking liquidity and bank runs}}$

2 agents: households and banks

2 assets: durable asset (capital) and non-durable commodity.

$k^b_t + k^h_t = 1$ Total capital held by banks and households respectively

Banks are professionals and they can intermediate capital for households.

If bank makes investment:

$$\begin{array}{ccl} k^b_t & \rightarrow & \begin{cases} z_{t+1}\cdot k^b_t & \text{output} \\ k^b_t & \text{capital} \end{cases} \\ t & & t+1 \end{array}$$

If household holds capital directly:

$$\begin{array}{ccl} \left\{ \begin{matrix} k^h_t \\ \frac{1}{2}(k^h_t)^2 \end{matrix} \right\} & \rightarrow & \begin{cases} z_{t+1}\cdot k^h_t & \text{output } \cancel{\text{cap}} \\ k^h_t & \text{capital} \end{cases} \\ t & & t+1 \end{array}$$

Households are less efficient investors, so they face ~~manage~~ management cost: $f(k_t) = \frac{1}{2}(k^h_t)^2$

Households both consume and save. They can save in form of bank deposits or by holding capital directly.

Return to household from bank deposit

$$R_{t+1} = \begin{cases} \bar{R}_{t+1} & \text{if no bank run (households expect to get it)} \\ x_{t+1}\cdot \bar{R}_{t+1} & \text{if there's a bank run.} \end{cases}$$

$\bar{R}$ = promised return, x_{t+1} = share of promised return ~~$U_t = \Sigma \ln C_t$~~ $U_t = \sum_{j=0}^{\infty} \beta^j \ln C^h_{t+j}$

Households are risk-averse and seek to maximize expected utility subject to budget constraint:

$$\underset{\uparrow \text{ consumption}}{C^h_t} + \underset{\text{deposit}}{D_t} + \underbrace{Q_t k^h_t}_{\text{own investment}} + f(k^h_t) = \underbrace{z_t \cdot w^h}_{\text{endowment}} + \underbrace{(Q_t + z_t) k^h_{t-1}}_{\text{return own investment,}}$$

FOC for direct capital holding:

$E_t[\Lambda_{t,t+1} R^h_{t+1}] = 1$, where $\Lambda_{t;t+1} = \beta^j \frac{C_t}{C_{t+j}}$

$R^h_{t+1} = \frac{z_{t+1} + Q_{t+1}}{Q_t + \alpha k^h_t}$ – Return on direct capital holding

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Banks are risk-neutral and ~~see~~ ~~have~~ enjoy utility of consumption when exiting:

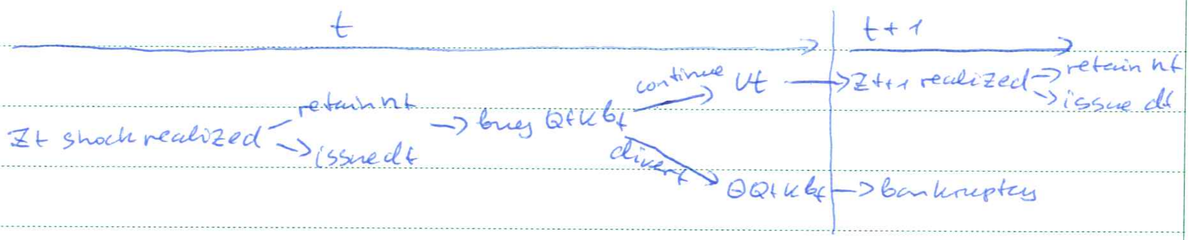
$$V_t = \sum_{j=1}^{\infty} \beta^j (1-\delta) \delta^{i-1} C^b_{t+i} = \sum_{j=1}^{\infty} \beta^j (1-\delta) \delta^{i-1} n_{t+i}$$

where δ = survival probability

$(1-\delta)\delta^{i-1}$ = probability of exiting before t+1.

~~balance sheet constraints~~

Let's look at banks behaviour,



After productivity shock Z_t is realized:

1) Net worth of continuing banks:

$n_t = (Z_t + Q_t) K^b_{t-1} - R d_{t-1}$

2) Net worth of exiting banks will be consumed:

$C^b_t = n_t$

3) Nett worth of entering banks: $n_t = w^b_t$
endowment

After banks have received deposits and purchased new capital ($Q_t K^b_t$) they have to choose whether they need to operate honestly (keep assets till payoffs in t+1 realized) and pay debt to depositors) or cheat - divert assets (sell assets for personal use). Banks can sell only a share, θ, of assets without being unnoticed. Cost of diverting assets is that households can stop rolling over deposits and force the bank into liquidation in the beginning of t+1.

So banks choice boils down to choosing between ~~its~~ its value V_t = PV(CF from operating honestly) and gains from diverting assets. Households know about this, so they won't put their deposits in the bank unless:

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$\boxed{V_t \geq \theta Q_t k^b_t}$ Incentive constraint,

The value of continuing bank:

$$V_t = E_t[\underbrace{\beta(1-\sigma)n_{t+1}}_{\text{if doesn't survive}} + \underbrace{\beta\sigma V_{t+1}}_{\text{it survives}}]$$

The net worth growth rate

$$\frac{n_{t+1}}{n_t} = \frac{(Z_{t+1}+Q_{t+1})k^b_t}{n_t} - \cancel{R_{dt}}\, R_{t+1}\frac{d_t}{n_t} =$$

$$= \frac{(Z_{t+1}+Q_{t+1})}{Q_t}\frac{Q_t k^b_t}{n_t} - R_{t+1}\frac{d_t}{n_t} =$$

$$= R^b_{t+1}\cdot\phi_t - R_{t+1}\frac{d_t}{n_t} = \underbrace{(R^b_{t+1}-R_{t+1})}_{\text{excess return from levered assets}}\phi_t + \underbrace{R_{t+1}}_{\text{return on safe assets}}$$

where

$\phi_t = \frac{\text{total assets}}{\text{net worth}}$ = leverage multiple

$R^b_{t+1} = \frac{Z_{t+1}+Q_{t+1}}{Q_t}$ = return on banks investment.

All banks and households are symmetrical, so:

1) $Q^b_t K^b_t = \phi_t N_t = N_t + D_t$

2) $N_t = \sigma((Z_t+Q_t)K^b_{t-1} - R_t D_{t-1}) + W^b$

3) $Y_t = C_t^h + C_t^b + f(K^h_t)$.

If productivity shock Z_t is large enough, but not too large, there could be to equilibria:

1) <u>normal equilibrium</u> where hoseholds trust the bank and keep rolling over deposits,

2) <u>Bank run equilibrium</u> where households don't trust each other and the bank and ~~start rol~~ stop rolling over deposits forcing bank into liquidation. There'll be a firesale of assets,

<u>The condition of the bank run equilibrium</u>

- comes from net worth; $n_t = (Z_t+Q_t)K^b_{t-1} - R_t D_{t-1}$
- Q^* = firesale price of assets

- If $\frac{(Z_t + Q_t^*)K^b_{t-1}}{R_t D_{t-1}} < 1$ there's a possibility of a bank run

Otherwise bank run equillibrium isn't possible:

~~R^b~~ Define: $x_t = \frac{(Z_t + Q_t^*)K^b_{t-1}}{R_t D_{t-1}} < 1$ The recovery rate.

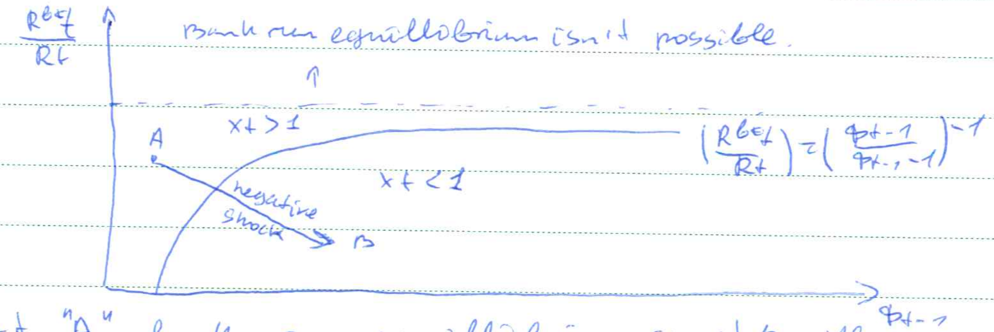
$$R^{b*}_t = \frac{Z_t + Q_t^*}{Q_{t-1}}$$

$$x_t = \frac{(Z_t + Q_t^*)Q_{t-1}K^b_{t-1}}{Q_{t-1} \quad R_t D_{t-1}} = \left(\frac{R^{b*}_t}{R_t}\right)\frac{\phi_{t-1}N_{t-1}}{D_{t-1}} = \left(\frac{R^{b*}_t}{R_t}\right)\frac{\phi_{t-1}}{D_{t-1}/N_{t-1}} =$$

$$= \left(\frac{R^{b*}_t}{R_t}\right)\frac{\phi_{t-1}}{\frac{Q_{t-1}K^b_{t-1} - N_{t-1}}{N_{t-1}}} = \left(\frac{R^{b*}_t}{R_t}\right)\frac{\phi_{t-1}}{\phi_{t-1}-1}$$

So recovery rate: $x_t = \left(\frac{R^{b*}_t}{R_t}\right)\frac{\phi_{t-1}}{\phi_{t-1}-1} < 1$

Where R^{b*}_t = return on banks assets in case of firesale,



At "A" bank run equillibrium isn't possible and depositors will always get the promised return,
At "B" bank run equillibrium is positive,
If shock Z_t is big enough and leverage is big enough, the Bank can end up in "B" and there'll be firesalle of assets,

Since all banks are symmetrical: if 1 bank collapses all banks collapse. And it will take time for new banks to appear and operate efficiently again. In that time households will need to ~~operate~~ invest on their own, and they aren't efficient,

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Eulers equation for households

$$R^{h*}_{t+1} = \frac{~~Q~~ Z_{t+1} + Q_{t+1}}{Q^*_t + k^h_t}$$

$$E[u_{t,t+1}(Z_{t+1} + Q_{t+1})] = Q_t^* + k^h_t$$

$$E_t[u_{t,t+1}(Z_{t+1} + u_{t+1,t+2}(Z_{t+2} + Q_{t+2})) - \delta k^h_t) = Q^*_t + k^h_t$$ so

$$Q_t^* = E_t \sum_{j=1}^{\infty} u_{t,t+j}(Z_{t+j} - \delta k^h_{t+j}) - \delta k^h_t$$ The firesale price of assets.

So ~~if~~ the longer time it takes for the banking sector to recover, the lower the firesale price.

So in this model banks are constrained by their networth (banks can't operate if $n_t < 0$) through incentive constraint. Their net worth depends on the firesale price of assets which in it's turn depends on liquidity on the markets. Liquidity on the markets can change fast in case of a bank run. Banks considered in the model are shadow banks: ~~be~~ completely unregulated, having short-term liabilities and long-term partially illiquid assets, and because of that being a potential subject to a bank run. If bank run occurs to one bank, the whole system collapses because banks are symmetrical. The possibility of a bank run is high in ~~crisis times~~ recession times.

3) $\boxed{\text{The model of shadow banking}}$

- investors (~~bank~~ households, pension funds) ~~households~~: have endowment w and are infinitely risk averse
- intermediaries (shadow banks) have endowment: w int.

Only intermediaries have access to projects - mortgages:

$I_{Hj} \leq 1$ with return R → Riskless mortgages, in limited supply

$$I_{Lj} = \begin{cases} A I_{Lj} & \text{with probability } \pi_w \\ 0 & \text{with probability } (1-\pi_w) \end{cases}$$ → Risky projects, unlimited supply. ~~depen~~ Returns depend on state of economy and whether intermediary ~~investor~~ is lucky.

$$w = (\underset{\substack{\uparrow \\ \text{growth}}}{g}, \underset{\substack{\uparrow \\ \text{downturn}}}{d}, \underbrace{r}_{\text{recession}})$$

π_w = probability of being lucky.

To fund these projects banks issue commercial paper (CP):
CP: $r \geq 1$ and promise it to be riskless.

Net return to banks

1) Gross payoff on riskless projects:

$$\underbrace{RI_{Hj}}_{\text{return on investment}} + (R - \underset{\substack{\uparrow \\ \text{price}}}{p_H})\underbrace{(S_{Hj} - T_{Hj})}_{\text{net market purchase}}$$

2) Gross payoff on risky projects:

$$\text{~~RH~~}\ \underbrace{E_w(\pi_w)A I_{Lj}}_{\text{uncertain payoff on risky projects}} + (E_w(\pi_w)A - \underset{\text{price}}{p_L})\underbrace{(\underset{\text{aggregate risk}}{S_{Lj}} - \underset{\text{aggregate + idiosyncratic risk}}{T_{Lj}})}_{\text{net market purchase}}$$

3) Cost of funding to substract:

$$I_{Hj} + I_{Lj} + (\cancel{\ \ })(1-r)D_j - w_{inf}.$$

Since intermediaries are risk-neutral they seek to maximize their expected return subject to budget constraints:

a) Funding constraint: $\underbrace{I_{Hj} + I_{Lj}}_{\text{investment}} + \underbrace{p_H(T_{Hj} - S_{Hj}) + p_L(T_{Lj} - S_{Lj})}_{\text{net market purchase}} \leq \underbrace{w_{int} + D_j}_{\text{own wealth + riskless debt.}}$

b) Safe debt constraint: $rD_j \leq R(I_{Hj} + T_{Hj} - S_{Hj}) + \pi_r A T_{Lj}$

So intermediaries can use as collateral on their riskless debt: returns on safe projects + returns on securitized projects. ~~The~~ They can't use return on risky projects as collateral because it includes idiosyncratic risk and investors are infinitely risk averse.

c) Feasibility of securitization constraint

$S_{Lj} \leq I_{Lj}$, $S_{Hj} \leq I_{Lj}$ - Intermediaries can't securitize more than they invest.

Leverage at securitization at $t=0$

Lemma 1: ~~was~~ intermediaries won't securitize riskless projects since they're already risky so $S_{Hj} = T_{Hj} = 0$. because ~~of~~ safe debt constraint they will securitize risky projects which they can trade only with each other

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since investors are risk averese, so $S_{Lj} = T_{Lj} = 0$,

Funding constraint becomes: $I_{Hj} + I_{Lj} \leq w_{int} + D_j$

Safe debt constraint becomes: $r D_j \leq R I_{Hj} + \pi_r A T_{Lj}$

- what securitization does:
  - boosts leverage
  - helps diversify idiosyncratic risk.
- Intermediaries securitize not because they won't to diversify risk, but because it's the only way to get funds from investors when it's not enough riskless projects.

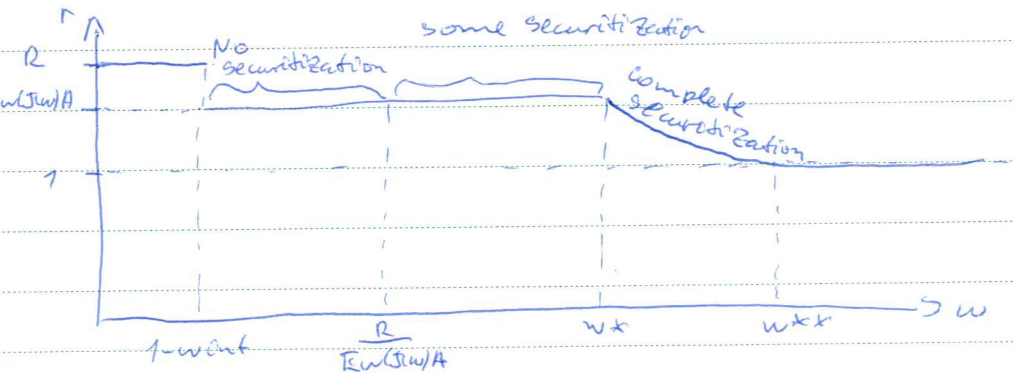
Intermediaries pursue a carry trade: $\underbrace{E_w(\pi_w)A}_{\text{excess return on securitized risky projects}} - \underbrace{r}_{\text{return on CP}}$

Given risk-neutral intermediaries and risk averse investors, intermediaries benefit a lot from the carry trade.

Securitization is useful ~~becau~~ if all agents recognize aggregate tail risks because it boosts leverage and helps diversify away idiosyncratic risk even for intermediaries. If aggregate tail risk is recognized the shadow banking system is very stable and investors ~~all~~ always get the promised return, however securitization makes banks interconnected trough markets (because they share the same risk) and increases their exposure to aggregate risk. → financial fragility

If tail risk is ignored, in recession times, the whole system will collapse because banks are symmetrical and interconnected with each other.

The equilibrium with recognized tail risk



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a) $w + w_{int} \leq 1$ Investors wealth is so low that intermediaries fund only riskless projects, No securitization,

$I_{Hj} = w + w_{int}$, $I_{Lj} = 0$, $S_{Lj} = T_{Lj} = 0$, $r = R$

b) $w \in \left(1 - w_{int}, \frac{R}{E_w(\pi_w)A}\right)$ - Investor wealth is higher so intermediaries fund some risky projects, but return on riskless projects is sufficient to guarantee debt, No securitization

$I_{Hj} = 1$, $I_{Lj} = w + w_{int} - 1$, $S_{Lj} = T_{Lj} = 0$, $r = E_w(\pi_w)A$

c) $w \in \left(\frac{R}{E_w(\pi_w)A}, w^*\right)$ - Investor wealth is higher so intermediaries can't guarantee CP by only riskless projects -> start securitizing.

$I_{Hj} = 1$, $I_{Lj} = w + w_{int} - 1$, $S_{Lj} = T_{Lj} \in (0, I_{Lj})$, $r = E_w(\pi_w)A$

As long as

$$w \leq w^* = \frac{R + \pi_r A(w_{int} - 1)}{E_w(\pi_w)A - \pi_r A}$$ => R, w_{int} = safety cusions, make sure that intermediarys doesn't lever up too much.

d) $w \in (w^*, w^{**})$ - Investor wealth is so high that full securitization is needed, ~~to a~~ To absorb all investor wealth interest rates have to fall;

$$r = \frac{R + \pi_r A(w + w_{int} - 1)}{w}$$

$$\frac{dr}{dw} < 0.$$

The results

Investors always get: $rD_j = RI_{Hj} + \pi_r A S_{Lj}$

Lucky intermediaries get: $RI_{Hj} + \pi_w A S_{Lj} + A(I_{Lj} - S_{Lj})$

Net : $\underbrace{(\pi_w - \pi_r)AS_{Lj}}_{=0 \text{ in the worst case}} + \underbrace{A(I_{Lj} - S_{Lj})}_{\geq 0}$

Unlucky intermediaries get: $RI_{Hj} + \pi_w A S_{Lj} + 0(I_{Lj} - S_{Lj})$

Net: $\underbrace{(\pi_w - \pi_r)}_{\geq 0} AS_{Lj}$

So everyone breaks even

The equillibrium with neglected tail risk

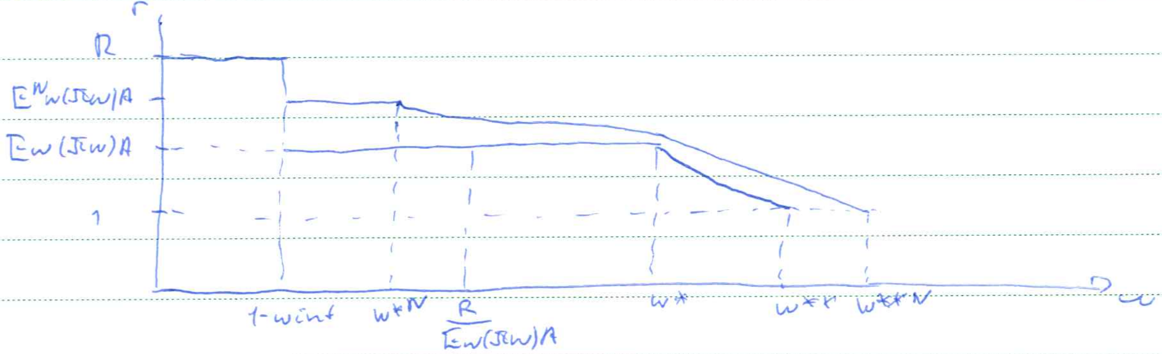
- people ignore the possibility of recession.
- people are overly optimistic:

$E^N_w(\pi_w) \geq E_w(\pi)$

$r^N \geq r$

$S^N_{Lj} \geq S_{Lj}$

They believe that CP will have higher return and invest more, so banks securitize more.



- So with neglected tail risk securitization starts earlier and lasts longer.
- If "d" everyone survives
- If "r" recession:

Investors get: $rD_j = RI_{Hj} + ~~\pi_d~~ \pi_d AS_{Lj}$

- Unlucky intermediaries get: $RI_{Hj} + \pi_r AS_{Lj} + A(I_{Lj} - S_{Lj})$

  Net: $(\pi_r - \pi_d)AS_{Lj} < 0$ So they don't survive.

- Lucky intermediaries get: $RI_{Hj} + \pi_r AS_{Lj} + A(I_{Lj} - S_{Lj})$

Net: $\underbrace{(\pi_r - \pi_d)AS_{Lj}}_{<0} + \underbrace{A(I_{Lj} - S_{Lj})}_{=0 \text{ if full securitization.}}$

So with neglected tail risk:

1) unlucky intermediaries collapse;
2) lucky intermediaries:
   a) If partial securitization $\left(\frac{I_{Lj}}{S_{Lj}} > 1 + \pi_d - \pi_r\right)$, can survive
   b) If full securitization → collapse. Then the whole system collapses and systematic risk becomes systemic.

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There are certain policies that can reduce the problems named in the named models:

1) The model of credit risk: government guarantees for farmers debt or subsidies to farmers (but not to gatherers) would reduce or remove collateral constraint, however to remove the incentive to cheat the regulation of farmers effort is needed. Real life example: deposit insurance.

2) The model of bank runs and the model of shadow banking: government regulation, deposit insurance, capital requirements.

In the model of bank runs capital requirements could reduse banks ~~sens~~ abiliaty to suffer losses. Then there would be less bank runs, however capital requirement would reduce banks' leverage, so investing wouldn't be as efficient.

Capital requirement int the model of shadow banking would increase intermediaries own wealth; so w_t = the time when they ~~hav~~ have to start securitization would be further away.

The model which explains the ~~event~~ Global financial crises best is the model of shadow banking. Before the crises: construction went wild and there were innovations in banking sector. Due to demographical changes a lot of wealth went to the reach people who spend little and ~~supply of~~ demand for riskless assets increased. To match that demand banks started securitizig. But the tail risk (the risk that morgage prices could drop on national level and there could be a deep recession) was ignored. So the whole system of shadow banking collapsed leading to a large slowdown in overall economy /recession

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