Q1

The equity premium is the excess return an investor is supposed to get if he invest in ~~riskless~~ risky assets ~~instead~~, like shares, instead of risk-free assets, like government bonds, e.g. 3 month treasury bills.

$$ERP = r_{e,t+1} - r_{f,t+1}$$

The equity premium puzzle (EPP) arises because standard theories can confirm that the ERP is positive, however, they ~~cannot show~~ have a problem with showing why it is so large in the real world. So it is not a qualitative problem, but a quantitative problem.

In generally the ERP is defined as (standard model with time-additive power utility function)

$$ERP = - \frac{cov(U'(C_{t+1}); r_{e,t+1})}{E(U'(C_{t+1}))}$$

Since the covariance is negative, the ERP becomes positive. Consumption rises if wealth rises (in case of a higher equity return), but since $U(C_{t+1})$ has a decreasing marginal utility the covariance is negative. So even ~~standard~~ simple standard models can explain a positive

NTNU

Kandidat nr. / Candidate no. 10017

Dato / Date 13.12.17 Side / Page 2

Emnekode / Subject: FIN3005

Antall ark / Number of pages: 22

Denne kolonne er forbeholdt sensor

This column is for external examiner

(X) → see page 8

equity risk premium.

The Lucas Tree model (LTM) is a model without a labor market or production. Every household is endowed with one ~~unit~~ tree which can be seen as equity. The only way of saving in this model is to purchase a share of a tree. The trees have fruits which can be seen as dividends. The households in the model are all equal. Every household has the same preferences and same constraints, so in equilibrium there is no borrowing or lending. The forces of demand and supply make sure that in equilibrium a tree or ~~equit share price~~ equity price is established at which all households are happy to just hold the amount of trees they started with. Furthermore, the forces of demand and supply make ~~sure~~ sure there is a risk-free rate at which every household is happy not to borrow or to lend.

The variables in this model are

consumption: c_t consumption growth: $\frac{c_{t+1}}{c_t} = x_{t+1}$

dividends: y_t dividend growth: $\frac{y_{t+1}}{y_t} = z_t$

In the equilibrium the consumption will be equal to dividends.

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In this model it is also reasonable to assume that asset prices are proportional to dividends, because dividends are the only source of income in this model. So they are the only source of value to the shareholders.

conjecture: $P_t = w \cdot \varphi_t$

Under the assumption of expected power utility the Euler equation ~~can be~~ ~~is~~ for this model is

$$U'(C_t) = \beta E[R_{e,t+1} U'(C_{t+1})]$$

with $U(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma}$

Plugging this into the Euler equation

$$C_t^{-\gamma} = \beta E[R_{e,t+1} C_{t+1}^{-\gamma}]$$

Kandidat nr. / Candidate no. 10017

Dato / Date 13.12.17 Side / Page 4

Emnekode / Subject: FIN 3005

Antall ark / Number of pages: 22

Denne kolonne er forbeholdt sensor

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the ~~asse~~ risky aset return ~~can be~~ is defined as

$$R_{e,t+1} = \frac{P_{t+1} + Y_{t+1}}{P_t}$$

inserting the conjecture : $P_t = w \cdot Y_t$

$$R_{e,t+1} = \frac{w \cdot Y_{t+1} + Y_{t+1}}{w \cdot Y_t} = \frac{1+w}{w} \cdot \frac{Y_{t+1}}{Y_t} = \frac{1+w}{w} \cdot z_{t+1}$$

Inserting this into the Euler equation

$$1 = \beta E\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \cdot \frac{1+w}{w} \cdot z_{t+1}\right]$$

$$1 = \beta E\left[X_{t+1}^{-\gamma} \cdot \frac{1+w}{w} \cdot z_{t+1}\right]$$

solving for w

$$w\left(1 - \beta E\left[X_{t+1}^{-\gamma} \cdot z_{t+1}\right]\right) = \beta E\left[X_{t+1}^{-\gamma} \cdot z_{t+1}\right]$$

$$w = \frac{\beta E\left[X_{t+1}^{-\gamma} \cdot z_{t+1}\right]}{1 - \beta E\left[X_t^{-\gamma} \cdot z_{t+1}\right]}$$

Inserting this into

$$R_{e,t+1} = \frac{1+w}{w} \cdot z_{t+1}$$

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$$R_{e,t+1} = \frac{1 + \frac{\beta E[x_{t+1}^{-\gamma} \cdot z_{t+1}]}{1-\beta E[x_{t+1}^{-\gamma} \cdot z_{t+1}]}}{\frac{\beta E[x_{t+1}^{-\gamma} \cdot z_{t+1}]}{1-\beta E[x_{t+1}^{-\gamma} \cdot z_{t+1}]}} z_{t+1}$$

$$= \frac{\frac{1-\beta E[x_{t+1}^{-\gamma} \cdot z_{t+1}] + \beta E[x_{t+1}^{-\gamma} \cdot z_{t+1}]}{1-\beta E[x_{t+1}^{-\gamma} \cdot z_{t+1}]}}{\frac{\beta E[x_{t+1}^{-\gamma} \cdot z_{t+1}]}{1-\beta E[x_{t+1}^{-\gamma} \cdot z_{t+1}]}} z_{t+1}$$

$$= \frac{z_{t+1}}{\beta E[x_{t+1}^{-\gamma} \cdot z_{t+1}]}$$

Taking expectations

$$E[R_{e,t+1}] = \frac{E[z_{t+1}]}{\beta E[x_{t+1}^{-\gamma} z_{t+1}]}$$

Taking logs

$$\ln E[R_{e,t+1}] = \mu_z + \frac{\sigma_z^2}{2} - \ln\beta - \left(-\gamma\mu_x + \mu_z + \gamma^2\frac{\sigma_x^2}{2} + \frac{\sigma_z^2}{2} - \gamma\sigma_{xz}\right)$$

$$= -\ln\beta + \gamma\mu_x - \gamma^2\frac{\sigma_x^2}{2} + \gamma\sigma_{xz}$$

Kandidat nr. / *Candidate no.* 10017
Dato / *Date* 13.12.17 Side / *Page* 6
Emnekode / *Subject:* FIN 3005
Antall ark / *Number of pages:* 22

Denne kolonne er forbeholdt sensor
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~~The risk free rate becomes~~

Since the Euler equation has to hold for ~~the~~ risk free rate as well, it ~~becomes~~ becomes

$$1 = \beta E[X_{t+1}^{-\gamma} \cdot R_{f,t+1}]$$

Solving for $R_{f,t+1}$

$$R_{f,t+1} = \frac{1}{\beta E[X_{t+1}^{-\gamma}]}$$

Taking logs

$$\ln R_{f,t+1} = \ln 1 - \ln\beta + \gamma\mu_x - \gamma^2\frac{\sigma_x^2}{2}$$

Thus the ERP becomes

$$\ln E[R_{e,t+1}] - \ln E[R_{f,t+1}]$$

~~$= \mu_z + \frac{\sigma_z^2}{2} - \ln\beta + \gamma\mu_x - \mu$~~

$$= -\ln\beta + \gamma\mu_x - \gamma^2\frac{\sigma_x^2}{2} + \gamma\sigma_{xy} - \left(-\ln\beta + \gamma\mu_x - \gamma^2\frac{\sigma_x^2}{2}\right)$$

$$= \gamma\sigma_{xy}$$

So the ERP is the RRA times the covariance between consumption ~~and~~ growth and dividend growth. Because in equilibrium consumption equals dividends, this can also be ~~wr~~ written as

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$ERP = \gamma \sigma_x^2$

So the ERP is the product of the RRA and the variance of the consumption growth.

When plugging in real world data for consumption growth, a γ of 45.9 (for Mehra's calculations) is needed. This rate of risk aversion is ~~way~~ way higher than ~~at~~ estimates calculated by ~~micro~~ micro-economic experiments. ~~Usually~~ A value of ~~aroun~~ around 5 to 10 would be reasonable. So the ~~proble~~ problem in the LTM is that it needs an abnormally large value of risk aversion to hold for real world data. This is the ~~Even though people~~ so called Equity Premium ~~A further proble arises when looking at~~ Puzzle. A further problem arises when looking at the risk-free rate. For γ-values ~~that would aroun~~ around 40 it will get negative whereas for reasonable γ-values it will be way to large if one allows for a consumption growth

In case of no consumption growth

$\ln R_{f,t+1} = -\ln \beta$ with $\beta = \frac{1}{1+\theta}$

$\theta \simeq 0.02$ a reasonable risk-free rate

$\ln R_{f,t+1} = 0.02$

But ~~not~~ when allowing for consumption growth μ_x and the variance ~~term~~ are multiplied by γ & $\gamma^2/2$ thus leading to the so-called Risk-free rate puzzle.

(X) for page 2

Even though people do not seem much risk-averse, they are willing to pay a lot (in terms of lower returns) to avoid risk.

NTNU

Kandidat nr. / *Candidate no.* 10017

Dato / *Date* 13.12.17 Side / *Page* 9

Emnekode / *Subject:* FIN3005

Antall ark / *Number of pages:* 22

Denne kolonne er forbeholdt sensor

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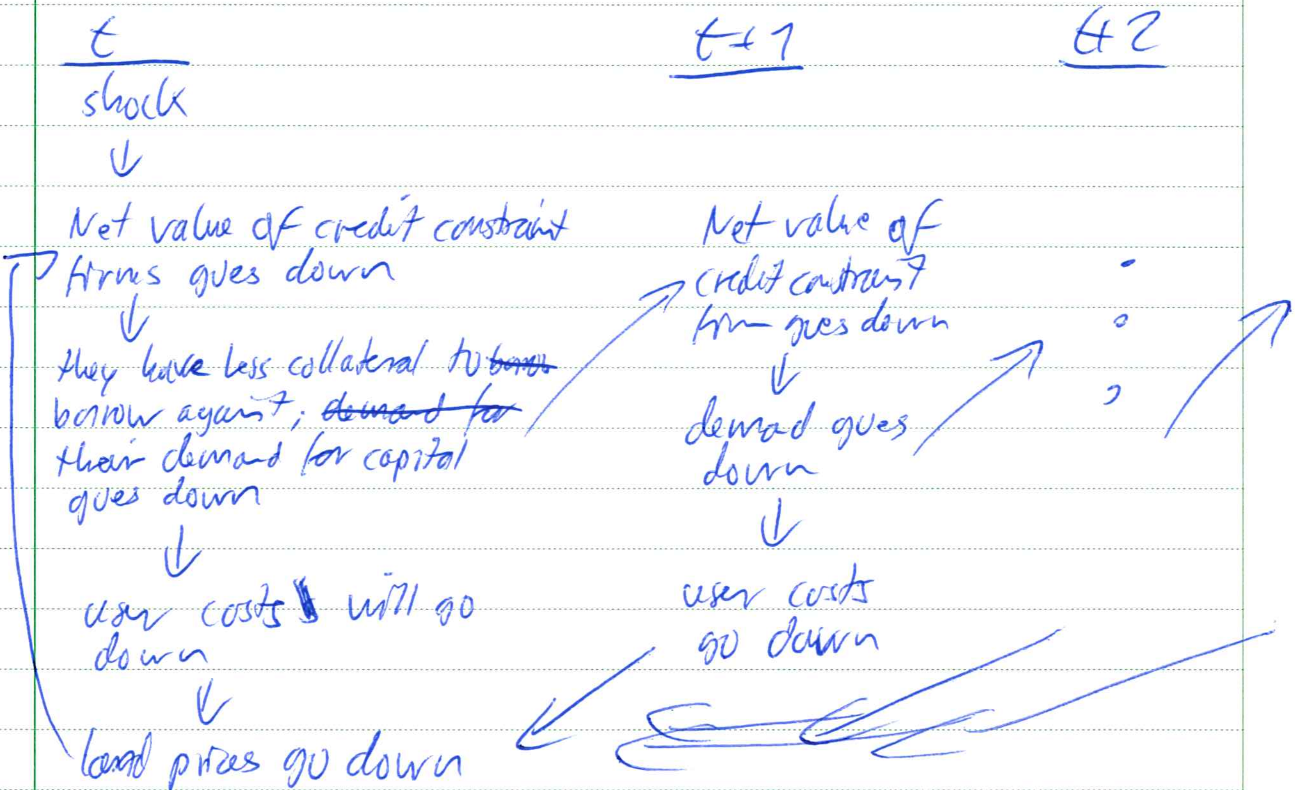
Q4

In the model by Kiyotaki and Moore capital (land) is used for production as well as for collateral when taking out a loan.
Some firms are credit-constraint, some are not.
If there is a productivity shock, it is amplified by this credit constraint.



There is a static and a dynamic multiplier effect.
The shock decreases the net value of credit constraint firms, so they can borrow less thus they demand less land. For the market to clear, the not constraint firms have to buy the land, but they only increase their demand if the user costs go

NTNU

Kandidat nr. / *Candidate no.* 10017

Dato / *Date* 13.12.17 Side / *Page* 10

Emnekode / *Subject:* FIN 3005

Antall ark / *Number of pages:* 22

Denne kolonne er forbeholdt sensor

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down. This is the case in every period. Because the ~~net value of a credit~~ demand of credit-constraint firms has ~~an~~ effect on their net value in the future and because the current price is determined by future user costs, todays price will go down. The static effect is the ~~effect~~ effect only in ~~per~~ the period the shocks happens, this is relatively small. The dynamic ~~is~~ ~~effect is~~ (intratemporal) / effect is intertemporal and because the whole future has an effect on todays' prices, this effect is large.

In the ~~mo~~ model by K&M there are 2 agents, farmers and gatherers. The farmers are ~~les~~ impatient whereas the gatherers are more patient. The farmers can use the land to produce more output, whereas the gatherers only pick the fruit that ~~grows~~ grows on the land. The land held by farmers is K, by gatherers ~~it~~ is K', K + K' = 1

Because the gatherers are more patient, they end up ~~being~~ the lenders, ~~the~~ the farmers are the borrowers. ~~But~~ The moral hazard in this model arises because farmers can just walk away from their land and abandon it. No one else can produce on the land, so ~~its~~ its just ~~worth~~ worth the price of the land. So farmers cannot borrow ~~ag~~ against their future income from growing plants, but only against the value of their land. Thus their collateral constraint is $R \cdot b_t \leq q_{t+1} \cdot K_t$

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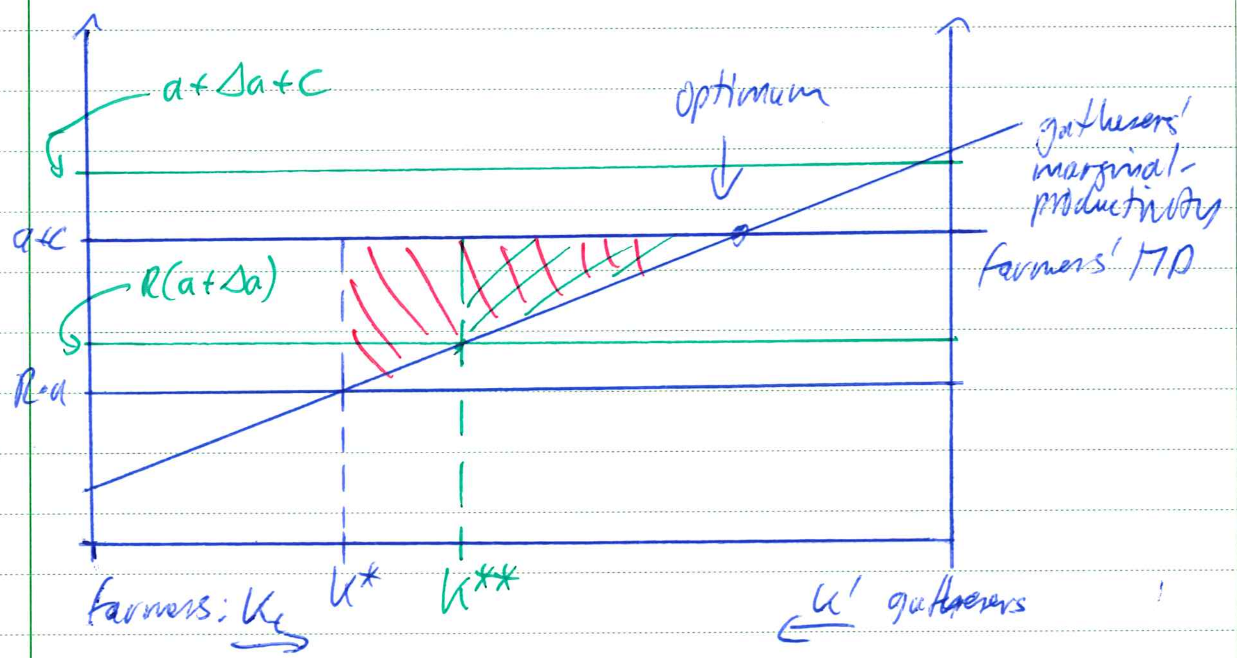
They cannot borrow more than their land is worth in the next period.

In equilibrium this leads to a situation where the farmers hold less capital than would be optimal. In the equilibrium their marginal productivity is greater than the gatherers' marginal productivity and thus a deadweight loss arises.



If a shock occurs, farmers' marginal productivity will rise and by ~~been~~ being amplified through the collateral constraint, they end up holding a lot more capital (K^{**}) than before. This decreases the DWL to ///.

~~To see how much the land holding changes through the effects, lets look at the change in~~

To see how much the ~~the~~ shock, amplified through the credit constraint, impacts the economy, lets look at the change in land holding by farmers and at ~~the~~

Kandidat nr. / Candidate no. 10017

Dato / Date 13.12.17 Side / Page 12

Emnekode / Subject: FIN3005

Antall ark / Number of pages: 22

Denne kolonne er forbeholdt sensor

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the change of ~~land prices~~ the land price; with and without the dynamic multiplier.

~~Land h~~ Change in land holding

with dynamic multiplier: $\hat{\kappa} = \frac{1}{1+\frac{1}{\eta}}\left(1+\frac{R}{R-1}\frac{1}{\eta}\right)\Delta \approx 26\Delta$

with only static: $\hat{\kappa} = \Delta$

when assuming an elasticity of $\eta \approx 1$ and a net return of 2% this leads to approximately ~~the values above above~~ to a 26 times higher change in land holding by the farmers through the dynamic effects.

~~Pa~~

The price change is also much larger ~~in case~~ due to the intertemporal effect

with dynamic m.: $\frac{1}{\eta}{}^{R}\Delta \approx \Delta$

with only static: $\frac{1}{\eta}\frac{R-1}{R}\Delta \approx 0{,}02\Delta$

So the price change is around 50 times higher ~~in case the~~ due to the dynamic effects, it changes by around the size of the initial shock.

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~~The dual role of land~~:

To summarize, the dual role of land, as collateral and a factor of production, can be described in the following way. A positive shock for example increases productivity, this increases the net worth of the credit constraint firm. This allows them to borrow more capital. This higher amount of capital can now be used in production but also again as collateral for more loans. This effect goes on and on, leading to a much higher effect than the initial shock.

The moral hazard keeps the economy from reaching its optimum. In equilibrium there is a deadweight loss since farmers' marginal productivity is greater than the gatherers'. But because of the farmers' incentive to default on their debt if it gets too high, their lending and borrowing is limited to $Rb_t \leq q_{t+1}k_t$. The government could now start to subsidize the farmers, by e.g. taxing the gatherers, or guarantee the farmers debt. This would decrease the DWL in equilibrium, but it would not solve the moral hazard problem. Only this time the farmers would walk away from the government. The moral hazard problem could, however, be solved by regulations of farmers' effort.

Kandidat nr. / *Candidate no.* 10017

Dato / *Date* 13.12.17 Side / *Page* 14

Emnekode / *Subject:* FIN 3005

Antall ark / *Number of pages:* 22

Denne kolonne er forbeholdt sensor

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Q5

Securitization is the pooling of risky assets, e.g. mortgages, to create a pool of (seemingly) riskless assets through diversification, e.g. mortgage backed securities.

Securitization can help to efficiently allocate risk by ~~eliminat~~ eliminating idiosyncratic risk; however, banks become equally exposed to aggregate risk. The model by Gentler, Schleifer & Vishny uses infinitely ~~risk~~ risk-averse households and risk-neutral banks as agents. The banks only securitize their assets to get a more ~~pre~~ predictable stream of safe income, so they can offer more safe assets to the households. Both agents have an endowment, banks ~~W_{int}~~ (or intermediaries) W_{int} and households W.

Kandidat nr. / Candidate no. 10017

Dato / Date 13.12.17 Side / Page 15 ~~16~~

Emnekode / Subject: FIN3005

Antall ark / Number of pages: 22

Denne kolonne er forbeholdt sensor

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The households only invest ~~1~~ in safe bank deposits yielding $r \geq 1$

Banks can invest in safe projects ~~(H) and~~ (I_H) and get a return $R >> 1$

~~They also also can~~ The amount of H-projects, however, is limited to 1. It can be seen as loans to wealthy people, and there is only a limited amount of these.

The banks also can invest in risky projects (I_L). These yield

$$\begin{cases} AI_L & \text{with probability } \pi_\omega \\ 0 & \text{with probability } (1-\pi_\omega) \end{cases}$$

π_ω is the share of "lucky projects". It is ~~the~~ idiosyncratic risk, because ~~none~~ noone knows ~~who~~ before, which the lucky projects are.

$\omega = \{recession, downturn, growth\}$ This is the ~~agre~~ aggregate risk. The share of lucky projects depends on the state of the economy $\pi_r < \pi_d < \pi_g$

Banks can now securitize their assets by selling off their risky projects and purchasing new diversified ~~ri~~ ~~riske~~ "riskless" projects.

The ~~return~~ gross return of safe projects is

$$R(I_H - S_H + T_H) + p_H(S_H - T_H)$$

return of holding safe assets plus return of trading

The gross return of risky projects

~~$R(I_L - S_L)$~~

$$(I_L - S_L)E(R_w)A + E(R_w)A \cdot T_L + p_L(S_L - T_L)$$

- $(I_L - S_L)E(R_w)A$: not securitized assets on the balance sheet → full risk
- $E(R_w)A \cdot T_L$: securitized risky projects → only aggregate risk
- $p_L(S_L - T_L)$: return of trading

The costs ~~of~~ are

$$rD + I_H + I_L + p_H(S_H - T_H) + p_L(S_L - T_L) - W_{int} - D$$

The bank is subject to 3 constraints

1) Funding constraint

$$I_L + I_H + p_H(T_H - S_H) + p_L(T_L - S_L) \leq W_{int} + D$$

They cannot spend more on investment than their own wealth plus deposits

2) Solvency in the worst case - constraint

$$rD \leq R \cdot (I_H - S_H + T_H) + R_r A T_L$$

In the worst case (a recession) the return to the banks investments has to be equal or greater than their obligation to depositors.

Kandidat nr. / Candidate no. 10017

Dato / Date 13.12.17 Side / Page 17

Emnekode / Subject: FIN3005 Antall ark / Number of pages: 22

Denne kolonne er forbeholdt sensor
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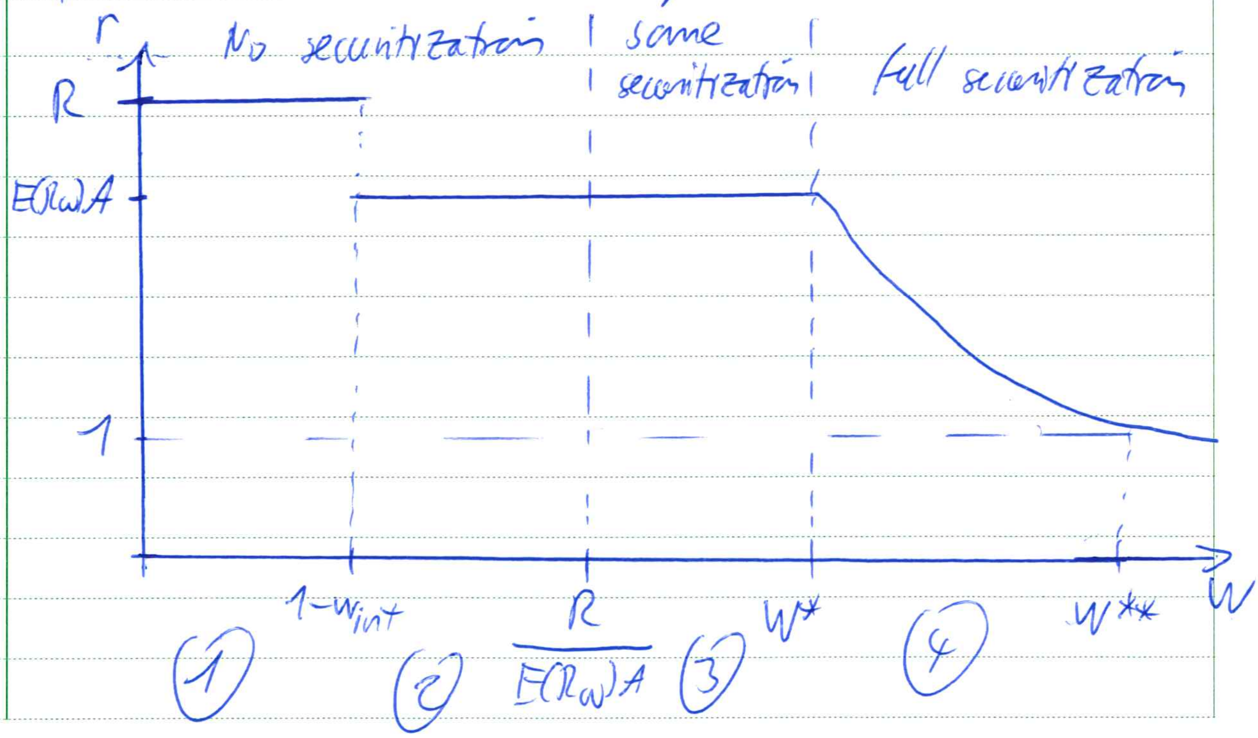
3) Feasibility of securitization
$S_L \leq I_L$ $S_H \leq I_H$
The bank cannot securitize more than they invested

If constraint (2) is not binding, there will be no securitization, $S_L = T_L = 0$. However, when it becomes binding the banks start to securitize so that $S_L = T_L > 0$.
Furthermore there is no need to securitize safe projects, so that $S_H = T_L = 0$

If all tail risk are taken into account, that means expectations are rational and a recession is seen as the worst possible state of the economy, the return to deposits in equilibrium is the following



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w can be seen as the demand for safe assets.

(1) Very low demand for safe assets. Banks only invest in H-projects and do not need to securitize. The return for the households is
$r = R$. $I_L = 0$; $0 \leq I_H \leq 1$ $S_L = T_L = 0$

(2) Demand for safe asset rises, but return of H-projects is still enough so that no securitization is needed. Banks start to invest in risky assets, return drops.
$I_L > 0$, $I_H = 1$, $S_L = T_L = 0$, $r = E(R_w) A$

(3) Demand for safe assets increases even further. Banks now need to start to securitize their risky projects.
$I_L > 0$, $I_H = 1$, $S_L = T_L > 0$, $r = E(R_w) A$
But the return for households is still the same

(4) The demand for safe deposits now is so high that the banks have to fully securitize all their risky projects. The return slowly drops down to $r = 1$ at w^{**}.
$I_L = T_L = S_L > 0$, $I_H = 1$, r goes to 1

Lets look at the outcomes if the tail risk is taken into account.

Households get their promised return

$$rD = R \cdot I_H + R_r A S_L$$

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Lucky banks get ~~a return~~

$A(I_L - S_L) + (R_p - R_w)AS_L \geq 0$

≥ 0 — If can at least get 0 in the case the bank securitizes everything

≥ 0 — can at least get 0 in case a recession happens

Unlucky banks

$0(I_L - S_L) + (R_r - R_w)AS_L \geq 0$

$= 0$ — don't get a return on unsecuritized risky projects

≥ 0

So ~~in~~ if tail risks are not neglected, households get their promised return and the banks survive. If the lucky banks securitized all their risky assets, there is no difference between unlucky and lucky banks anymore $I_L - S_L = 0$

In case ~~th~~ tail risk are neglected, everyone becomes more optimistic. A downturn is seen as the worst possible outcome instead of a recession.

$E^N(R_w) > E(R_w)$

$r^N > r$ depositors get a higher return

NTNU

Kandidat nr./*Candidate no.* 10017

Dato/*Date:* 13.12.17 Side/*Page:* 20

Emnekode/*Subject* FIN3005 Antall ark/*Number of pages:* 22

Denne kolonnen er forbeholdt sensor

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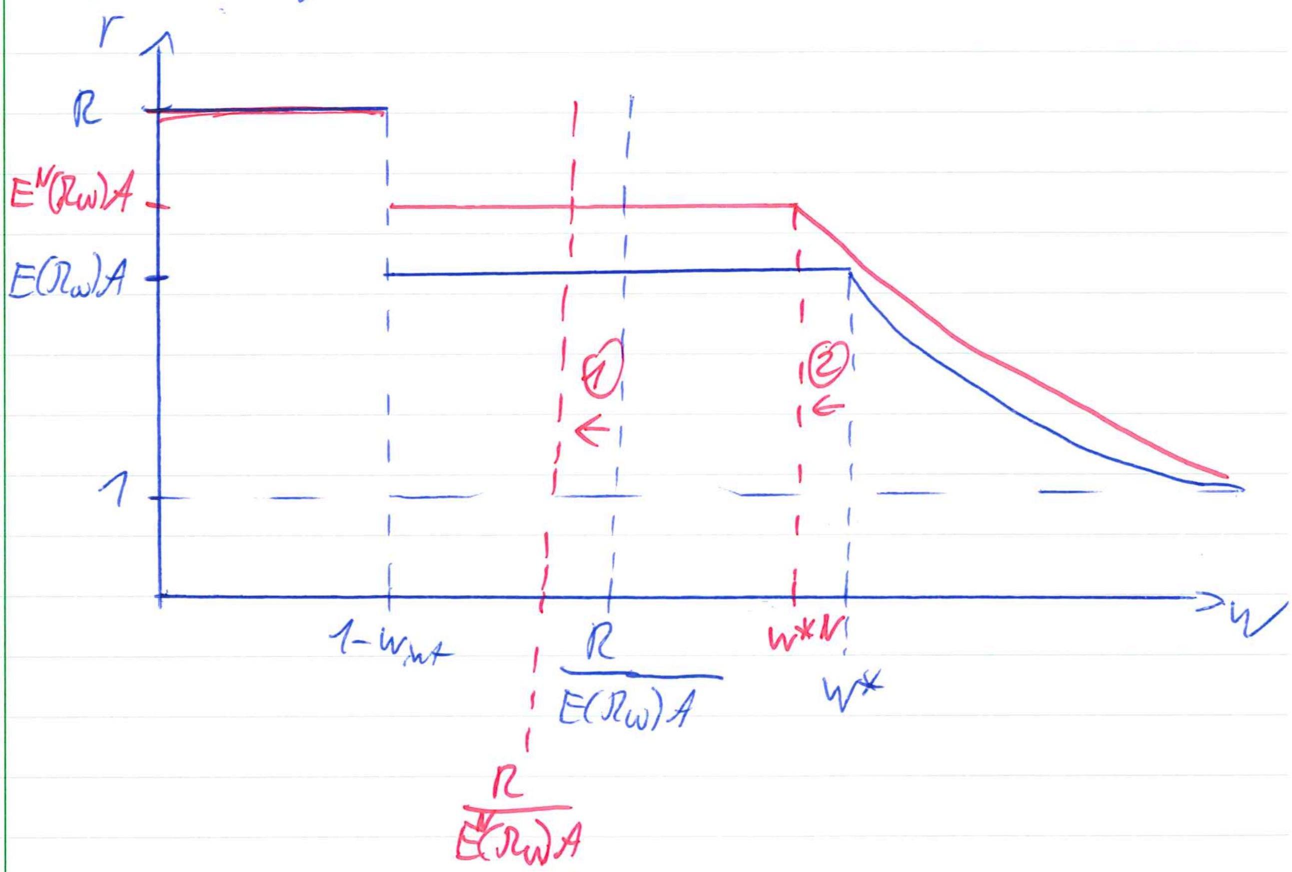
$D^N \geq D$ they may also deposit more

$S_L^N \geq S_L$ banks securitize more or at least the same amount

This leads to the following return to deposits in equilibrium in comparison to the case if they do not neglect tail risk.



So the return is ~~always~~ generally higher. Furthermore, banks start to securitize earlier (1) and also to fully securitize earlier (2).

The output changes in the following way

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Lucky banks

$$A(I_i^N - S_i^N) + AS_i^N(R_w^* - R_d) \geq 0$$

≥ 0 still cannot be smaller than 0

≥ 0 In case of a recession this becomes smaller than 0, since $R_r < R_d$

Unlucky banks

$$0(I_i^N - S_i^N) + (R_w - R_d)\, AS_i^N \geq 0$$

$= 0$ $\quad \geq 0$

If no recession occurs everything is fine, all banks at least survive. However, if a recession occurs unlucky banks go bankrupt. Lucky banks still have a chance to survive if they did not securitize all their assets. They survive if

$$\frac{I_i^N}{S_i^N} > 1 + R_d - R_r$$

The model makes clear that securitization can help to efficiently allocate risk. Through securitization banks are able to securitize away their idiosyncratic risk. However by doing so all banks become equally exposed to aggregate risk. This is not a problem if all tail risks are taken into account. In this case if a recession occurs all banks at least survive.

Kandidat nr. / *Candidate no.* 10017

Dato / *Date* 13.12.17 Side / *Page* 22 ~~22~~

Emnekode / *Subject:* ~~FIN~~ FIN3005

Antall ark / *Number of pages:* ~~22~~ 22

Denne kolonne er forbeholdt sensor

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But securitization becomes a problem if tail risks are neglected. By neglecting these risks, a recession can lead to a breakdown of the whole banking sector, because through securitizing all banks became equally exposed to aggregate risk and ~~if~~ since they are symmetrical, if one bank fails all banks fail. ~~Only~~ It is not possible to securitize away the aggregate risk. However if the ~~t~~ tail risk is not neglected this should not be a problem.