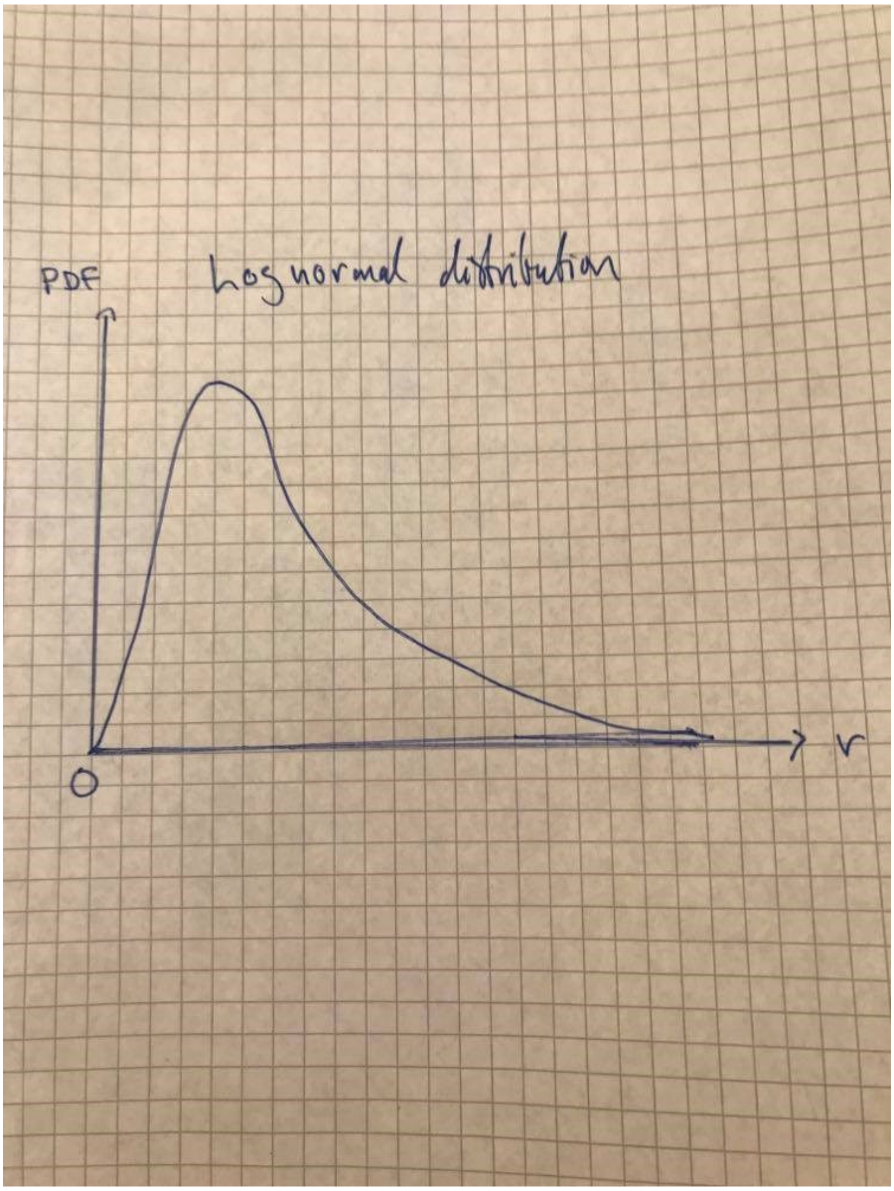
Task 1-a: Lognormal distribution

I chose to explain log-normal distribution in the context of returns, as it is most relevant in our case. When the instantaneous net return is normally distributed $r(t) \sim N(\bar{r}, \sigma^2)$. Then discrete gross return is log-normally distributed $lnR_{t+1} = r_{t+1} \sim N(\mu, \sigma^2)$. Where $\mu = \bar{r} - \frac{1}{2}\sigma^2$.



The gross return is the exponential of the normal variable, which is always non-negative. Hence, a gross return of zero is a net return of -100%. In other words, the log-normal distribution is handy for finance because returns can't be negative in gross terms. E.g. you can't lose more that you invest, which is true in the "normal" case of non-leveraged equity purchases.

Task 1-b: Dynamic programming and the Euler equation

Dynamic programming is about making the optimal decisions when the future is inherently uncertain, which also applies for the return on risky assets. I assume that the individual's income only derives from financial asset returns and that the individual has a dynamic constant relative risk aversion (CRRA) utility function like

$$U_t = \frac{c_t^{1-\gamma}}{1-\gamma} + E_t \sum_{s=1}^{\infty} \beta^s \frac{c_{t+s}^{1-\gamma}}{1-\gamma} = \frac{c_t^{1-\gamma}}{1-\gamma} + E_t \beta U_{t+1}$$

She will maximize U_t subject to the dynamic capital constraint (or future wealth)

$$A_{t+1} = R_{p,t+1}(A_t - C_t) = [w_t R_{e,t+1} + (1 - w_t) R_{f,t+1}](A_t - C_t)$$

Which simply is the amount invested today (e.g. the difference between current wealth and consumption) times the portfolio return, considering both equities and risk-free investments.

Now, if the future were known – she could decide how to optimally allocate resources from the present state to infinity (or death). But since future outcomes is not known, she decides what to consume now in the current time period, and decide upon consumption in future in each respective time period as they come. That is, she postpones any decisions that can be postponed – until they no longer can't. Why? Despite that future outcomes are uncertain; she can assume that future decisions will be just as rational as the current decision.

A value function $V_t(A_t)$ represent the utility value of this flexibility in decision-making, e.g. value today equals the current wealth defined as consumption in terms of power expected utility and discounted future values. The individual seek to maximize this value function

$$V_t(A_t) = \ max_{c_t, w_t} \left\{ \frac{c_t^{1-\gamma}}{1-\gamma} + \ \beta E_t V_{t+1}(A_{t+1} \right\}$$

By taking the first derivates of $V_t(A_t)$

$$\frac{\partial V_t(A_t)}{\partial c_t} = c_t^{-\gamma} - \beta E_t V'_{t+1}(A_{t+1}) \frac{\partial A_{t+1}}{\partial c_t} = c_t^{-\gamma} - \beta E_t V'_{t+1} R_{p,t+1} = 0$$

We can solve for the first order for consumption as

$$c_t^{-\gamma} = \beta E_t V'_{t+1}(A_{t+1}) R_{p,t+1}$$

Whereas by using the envelope theorem that the marginal value of wealth at t+1 becomes

$$V'_{t+1}(A_{t+1}) = c_{t+1}^{-\gamma}$$

Substituting this into the former equation, we manipulate the expression to the Euler equation

$$c_t^{-\gamma} = \beta E_t c_{t+1}^{-\gamma} R_{p,t+1}$$

$$\Rightarrow \beta E_t (c_{t+1}/c_t)^{-\gamma} R_{t+1} = 1$$

The first part on the left-hand side, $\beta E_t (c_{t+1}/c_t)^{-\gamma}$ is the marginal rate of intertemporal substitution. The Euler Equation intuitively shows that when the investor has already optimized, then the utility of investing an additional unit of currency for higher future consumption, has the same marginal utility as consuming it now.

Task 1-c: RRA, EIS, and their relationship

Relative Risk Aversion (RRA) describes an investor's change in behavior towards taking on more or less risky investments when she receives a marginal increase in wealth

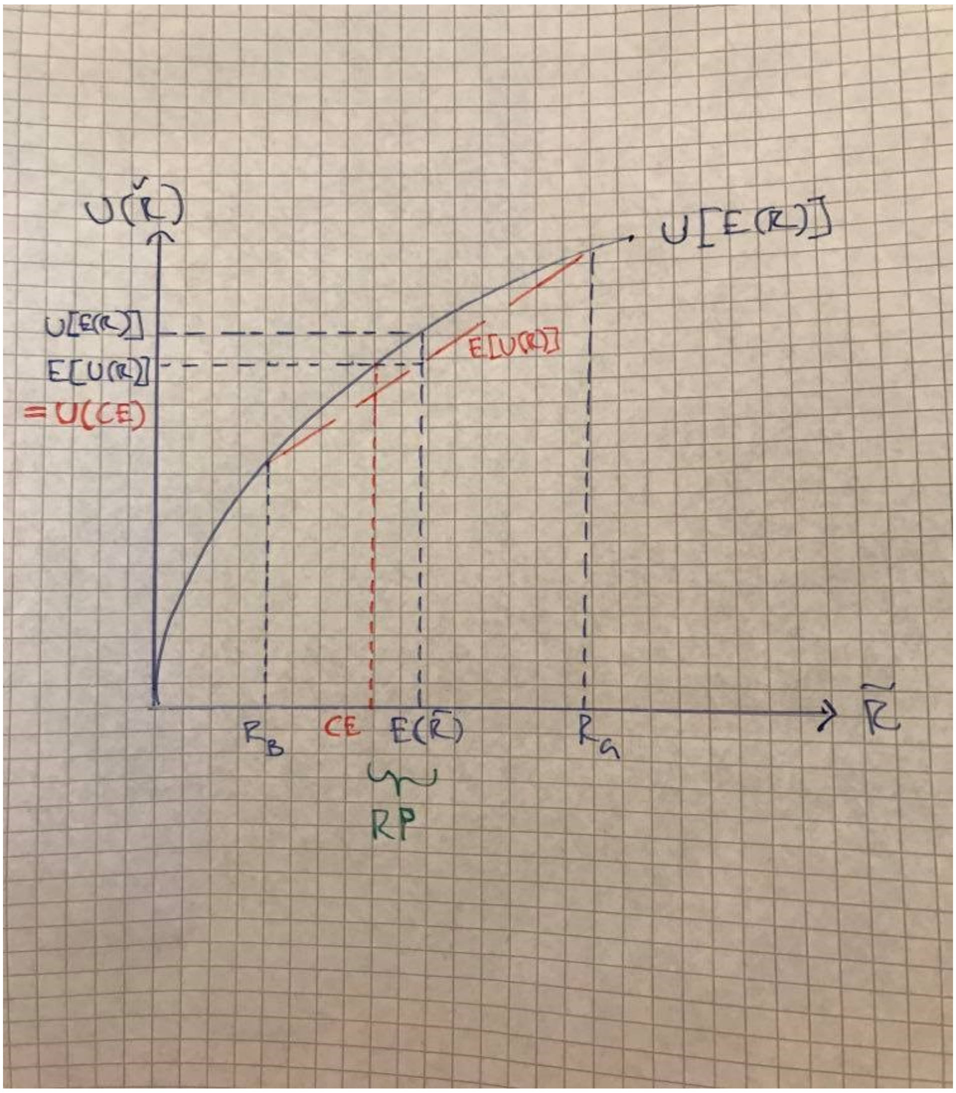
$$RRA = -c\frac{u''(c)}{u(c)}$$

The RRA may thus be positive, negative, zero or just a constant. In the case of power expected utility it is assumed that the investor exhibits (CRRA) constant relative risk aversion γ. The elasticity of intertemporal substitution EIS is one dividend by the RRA, $1/\gamma$. The EIS express how the investor will change her investment and consumption throughout time in response to changes in the risk-free rate. The EIS is sort of the time series finance version of MRS in basic microeconomics. The EIS under power expected utility preferences imply that the investor's aversion to unpredictable and predictable changes are equal.

However, when moving to Epstein-Zin preferences the relationship of RRA and EIS gets nuanced so that γ is the risk aversion to unpredictable changes, and δ is the aversion to predictable changes. If γ > δ, then the individual prefers an early resolution of future uncertainty. Vice versa if γ < δ. Under Epstein-Zin preferences 1/δ represents the intertemporal rate of substitution.

Oppgave 1-d: Certainty Equivalance

The intuitive explanation is that when facing a risky investment decision, the certainty equivalent (CE) is the risk-free alternative which yield the same level of utility. If the utility function U(R) of an investor is increasing, but diminishing, e.g. concave, the investor is said to be risk averse. That is, the first derivative U'(R) > 0 and the second derivative U'(R) < 0. Say that the risky bet has two outcomes, good R_G or bad R_B.



For any risky return along the utility function, the expected utility of return is higher than the utility of expected return: U[E(R)] > E[U(R)]. This is also known as Jensen's Inequality. We see from the graph that Certainty Equivalent is the utility of risk-free return which corresponds to expected utility of risky return. The difference is known as the risk premium $P = E(R) - CE(R)$.

Oppgave 1-d: Collateral in credit markets

Collateral is a claim on asset which creditor's require in order to lend money to borrowers. Why do creditors require collateral? That is due information asymmetry, because the lenders don't truly know the borrower's intention. There is a risk that the borrower takes the lender's money and run, or simply refusing to pay the money back. This is a problem labelled moral hazard. Hence, the lender require collateral. In the case where the borrower does moral hazard by refusing payback, the creditor can simply seize the asset(s) and sell it at the market.

There are also creditor's who don't require collateral, called "Forbrukslån" in Norwegian. As compensation due to the extra risk, the interest the borrower has to pay to the lender is of course very much higher than in the case of collateral.

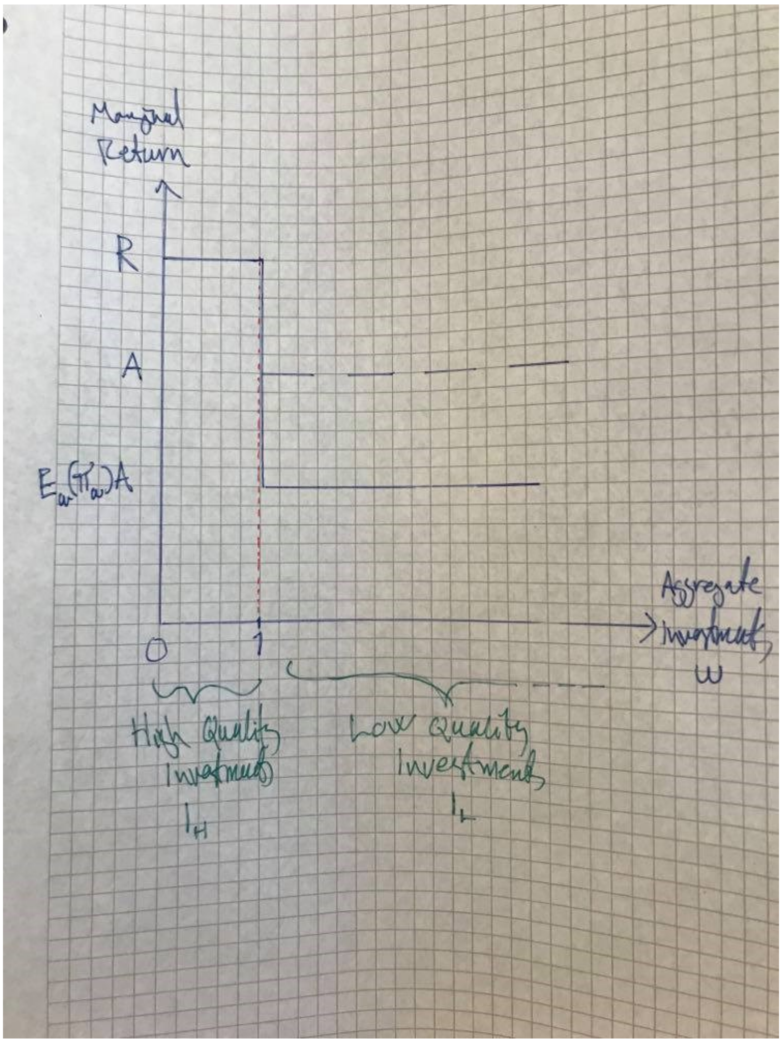
Furthermore, a model by Kyotaki and Moore show how creditors constrain credit as a function of the present asset value, and how this constraint exacerbates business cycles. This is also relevant for the real world. However, that goes beyond the scope of this task.

Oppgave 2-a: The GSV model

In GSV's paper "A Model of Shadow Banking" there are two agents, namely the financial intermediaries and the households (the clients who invest). All the households are infinitely risk averse, and thus for simplicity but without loss of generality we can assume that the number of households is one. Infinite risk aversion imply that they only want to invest at the risk-free rate r. The household indirectly invest by depositing their endowment of amount w at the financial intermediaries account for risk free return prospect. Hence, they get the same return in any state of the economy. They also possess no time preferences about present and future consumption. Thus, they're not interested in borrowing in order to consume early.

The financial intermediaries however are risk neutral. They are professional investors who seek to maximize their expected return, and don't have preferences regarding consumption now versus later. Their investment funds F thus consist of their own endowment w_{int} plus the deposits D by the households, so $F = w_{int} + D$. The financial intermediaries consider both high-quality investments I_H and low-quality investments I_L. The high-quality investments are in limited supply of $I_H \leq 1$, and they yield a gross return R X 1 > r. Naturally, the financial intermediaries will first invest their endowment in the high-quality projects, and then eventually move on to investing in low-quality projects as well. According to the projects' probability of succeeding, a π_ω share of the low-quality investments will be successful and yield gross rate return $A \times I_L$, while the other share $(1 - \pi_\omega)$ will be unsuccessful and yield 0 returns.

There are three possible states of the economy, which is either good, a downturn or a recession. Compactly these are expressed as $\omega\epsilon\{g, d, r\}$, and occur with probabilities $\pi_g > \pi_d > \pi_r$. These aggregate risks of the economy are what determines the probabilities low-quality projects being successful or not. The expression $E_\omega(\pi_\omega)A$ represents the expected return across the possible aggregate states of the economy.



The market price of high-quality projects is p_H. They are bought and sold between financial intermediaries. T_H is the quantity of purchases and S_H is the sales. We can now define the gross return for financial intermediaries from investing and trading high-quality projects as

$$H = R(I_H + T_H - S_H) + p_h(T_H - S_H)$$

Whereas the first term on the right-hand side is the gross return on risk-free projects, and the second term is the trading profits. Likewise, we can define the intermediaries' expected gross return on low-quality investment projects

$$L = E_{\omega}(\pi_{\omega})A * I_L + [E_{\omega}(\pi_{\omega})A - p_L] * (T_L - S_L)$$

Whereas the first term on the right-hand side is the expected return on risky investments exposed to both systematic and idiosyncratic risk, while the second term is the return on trading diversified securities who is only exposed to systematic risk. This expression can be reformulated to

$$L = E_{\omega}(\pi_{\omega})A(I_L - S_L) + E_{\omega}(\pi_{\omega})A * T_L + p_L (T_L - S_L)$$

Such that the first term is net investment in systematic and idiosyncratic risk, the second term is investments exposed to systematic risk only and the third term is net risk-free revenue from trading risky securities.

Cash outflows for financial intermediaries P thus consist of promised return r to the households' deposits D and money spent on investing in high- and low-quality projects

$$P = rD + I_L + I_H$$

We then may express the net expected cash flow for financial intermediaries as

$$NEC = F + H + L - P$$

As stated above, the financial intermediaries are profit seeking, which corresponds to maximizing net expected cashflow. This maximization problem is subject to three constraints

i) *The funding constraint*: the funds invested in different projects and net purchases can't exceed the households' deposits plus the intermediaries own endowment

$$I_H + I_L + p_H(T_H - S_H) + p_L(T_L - S_L) \leq F = D + w_{int}$$

ii) *The debt constraint*: the financial intermediaries must be able to pay back the risk-free return to the households, independent of which state the economy are in.

$$R(I_H + T_H - S_H) + \pi_r A * T_L \geq rD$$

iii) *The feasibility constraint*: the intermediary can't securitize more than they invest.

$$S_H \leq I_H, S_L < S_H$$

In equilibrium we thus have the following results …

- Households only deposit their money at the financial intermediaries as long as they guarantee a return r > 1. That is because none of the agents exhibit time preferences, and hence households don't borrow.

- There is no moral hazard. So, the financial intermediaries intend to honour the households' prospect of guaranteed return r.

- Given that the law of no arbitrage holds, the price of high-quality investments is $R/p_H = r$.

- Low-quality asset must yield a return which can satisfy the risk-free return on the households' deposits in any state of the economy. Then the price of such assets will be equal to the probability of a recession times return A. $p_L = \pi_r A \leq E_\omega(\pi_\omega)A$

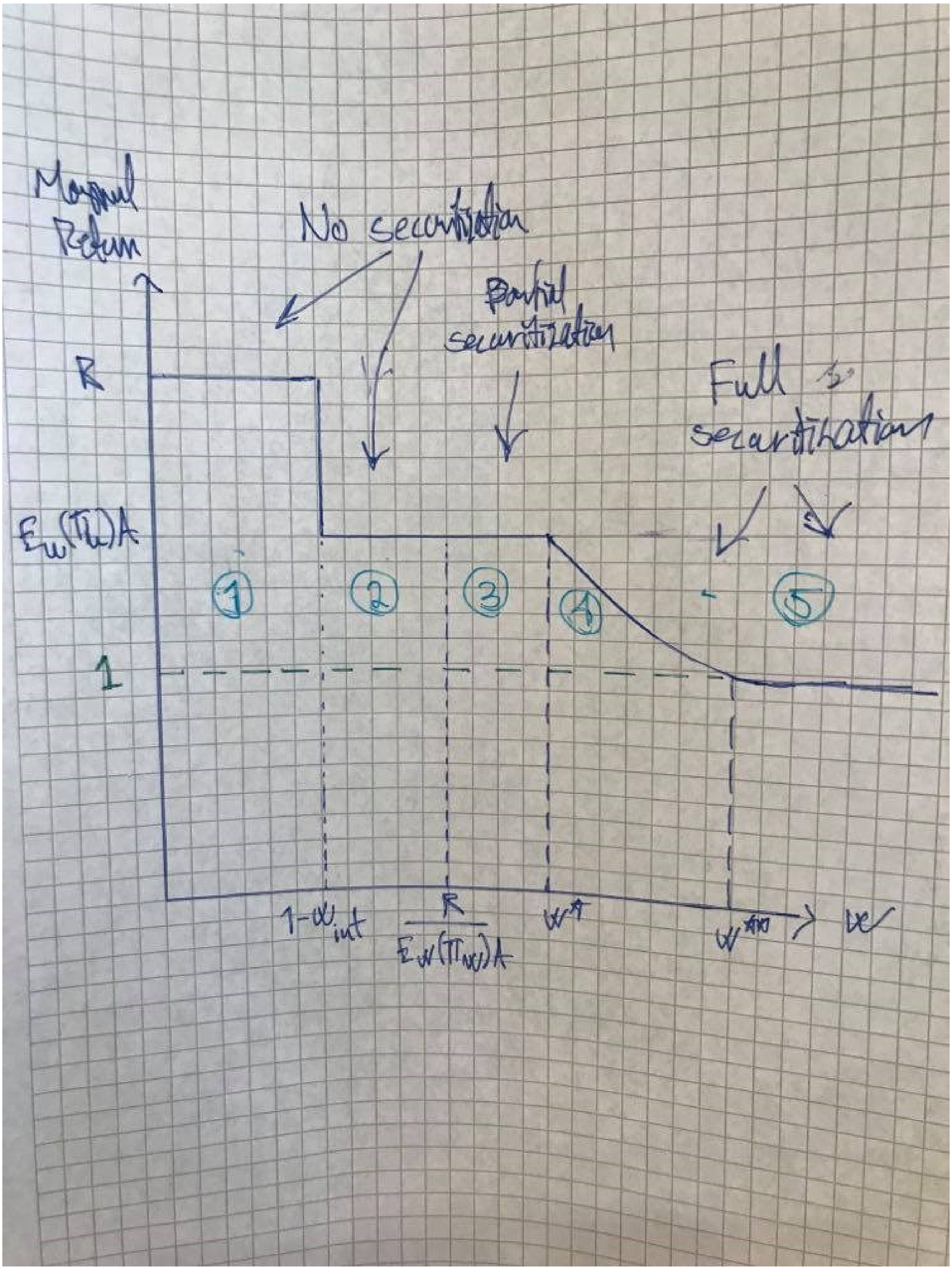
- High-quality projects do not need to be securitized: $T_H = S_H = 0$.

- The purchases and selling of securities between intermediaries must add up, $T_L = S_L$.

- If the funding constraint is binding, intermediaries securitize in order to meet the households' demand for deposits with risk-free return. The intermediary is effectively spreading risk by holding well-diversified securities which only contain systematic risk, and thus being able to accept more deposits than if no securitization.

- If, or when, the funding constraint is not binding, the demand for deposits is weak, and thus financial intermediaries will not need securitize low-quality projects in order to fulfil their obligations to the households.

So, how does all this affect the scale of securitization? Financial intermediaries will securitize the number of projects required to satisfy the households demand for risk-free return deposits corresponding to their endowment w.



As illustrated by the graph, there are five different situational cases according to the clients' demand for deposits

- Case 1: demand for deposits is low, such that $w \leq 1 - w_{int}$. This cause intermediaries to invest exclusively in high-quality projects, and the risk-free deposit rate equals the rate on high-quality prospects, r = R. Investments in high-quality projects is thus $I_H = w + w_{int} \leq 1$ and there is no investment in low-quality projects, $I_L = 0$. In this case the financial intermediaries do not securitize low-quality assets, so trading in such are zero, $T_L = S_L = 0$.

- Case 2: demand for deposits are higher than in case 1, so that $1 - w_{int} \leq w \leq R/E_\omega(\pi_\omega)A$. Demand for deposits does now exceeds what can be offered by high-quality projects alone, and this invest slightly in low-quality. So $I_H = 1$ and $I_L = w + w_{int} -1$. This cause the risk-free deposit rate to decrease from case 1 and be equal to the marginal return on low-quality projects $r = E_\omega(\pi_\omega)A < R$. Note that there is no securitization so far. That because the return from high-quality projects still manage to cover the intermediaries' obligations. So when no securitization, there is no trading: $S_L = S_T = 0$.

- Case 3: demand for deposits are now $\frac{R}{E_\omega(\pi_\omega)A} < w < w*$. The returns from high-quality investments will not be able to cover this alone, so the intermediaries will securitize some assets, and thus also trade some between themselves. $T_L = S_L > 0$. The risk-free deposit rate r is as in case 2, as long as the funding constraint is binding.

- Case 4: demand is $w^* < w < **$ and the intermediaries will securitize all low-quality projects available, hence $S_L = T_L = I_L$. The deposit rate decline further corresponding to the marginal return from low-quality projects.

- Case 5: $w > w^{**}$. Demand for risk-free deposit rate is higher than what is possible to offer. Then r = 1

As GSV shows, full securitization makes the economy fragile to systemic risk, but first let's look at the payoffs to households and intermediaries. Since households have deposited and invested all their wealth for at zero-risk rate of return, they get as what is promised …

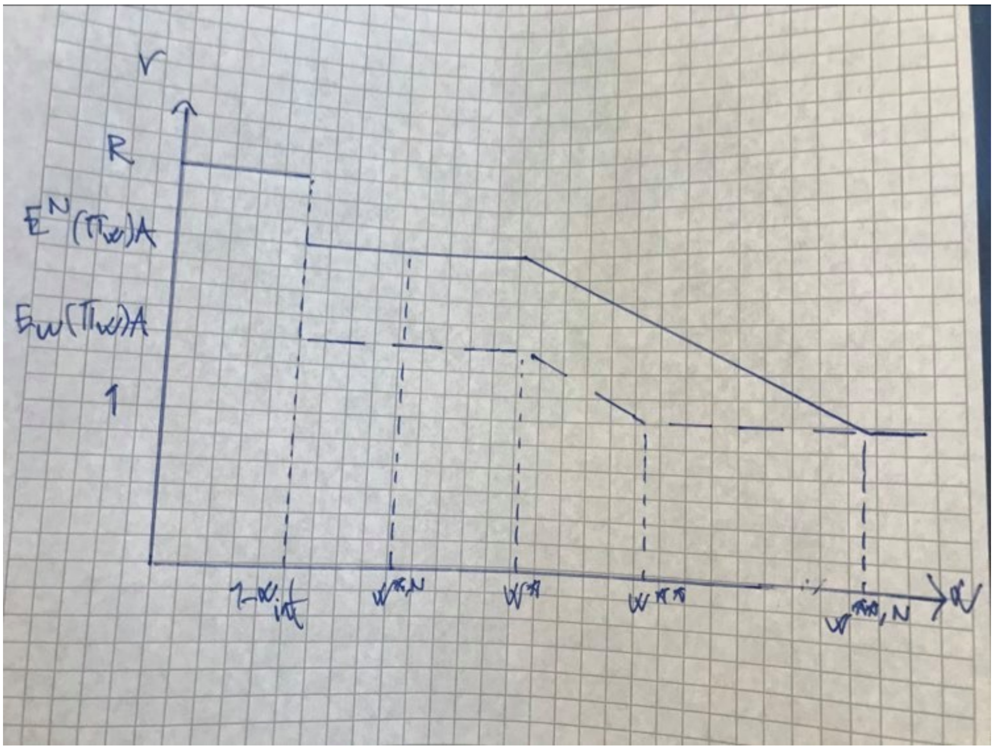
$$rw = rD = R^*I_H + \pi_r A^*S_L = R^*I_H + \pi_r A^*S_T$$

Which is the return from high-quality investments and returns from risky, securitized projects. We divide the intermediaries into two groups, those who are successful and those who is not. The successful intermediaries thus get $A(I_L - S_L) + (\pi_\omega - \pi_r)A * S_L \geq 0$. If $w < w^*$ securitization is not fulfilled, then intermediaries earn a positive return no matter the state of the economy. If $w > w^*$ so the we have full securitization, then the successful intermediaries will generate positive profits if a good or a downturn but break even if a recession. The unsuccessful intermediaries however earn $(\pi_\omega - \pi_r)A * S_L \geq 0$, which is positive in good times and downturns, but break even if a recession state. Although, they don't go bankrupt.

Now, consider a situation where aggregate tail risk are being neglected by both agents, such that the probability of recession is believed to be zero. Then expectations about returns, deposits and trading will be too optimistic, such that ...

$$r^N \geq r, \quad D^N \geq D, \quad S_L{}^N \geq S_L$$

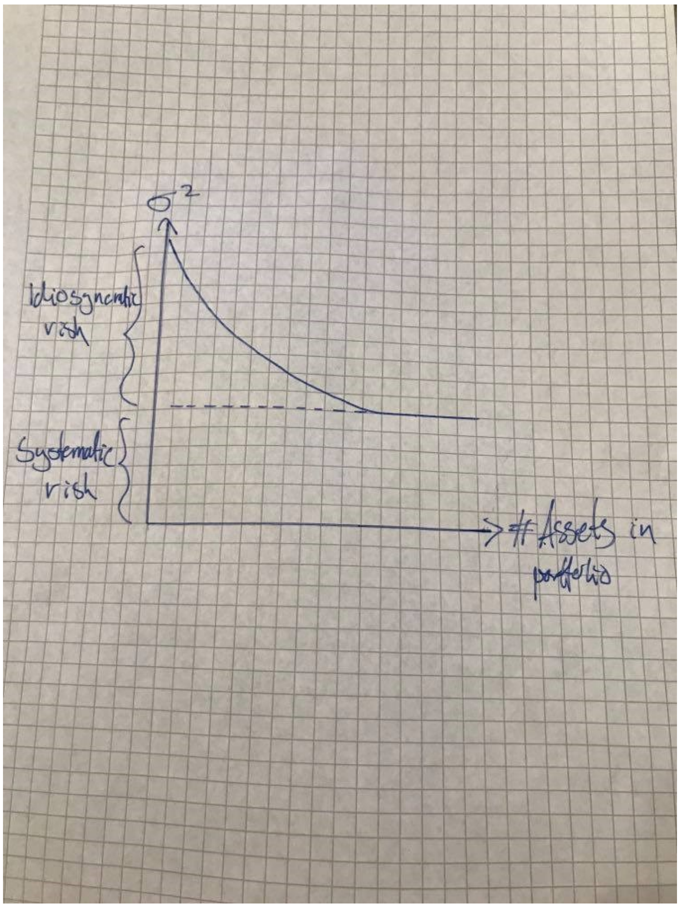
where subscript N is for neglect. And ultimately, they neglect the probability for being unsuccessful at the same time as everyone else, and then implicitly leveraging their positions by securitizing more low-quality investments than rationally ought to do. This causes new equilibriums, and for the cases in between $w < w_{int}$ and $w > w^{**,N}$, we see imbalance between promised return $E^N(\pi_\omega)A$ and rational expected returns $E_\omega(\pi_\omega)A$. That is, the difference between the dotted and the solid line.



When tail risk is neglected, the risk-free return prospect $r^N D^N$ is no longer guaranteed in reality. That's because if a recession occurs and securitization is completed, all intermediaries go bankrupt. How? Successful intermediaries have a payoff equal to $A * (I_L^N - S_L^N) + (\pi_\omega - \pi_d)A * S_L^N$ which is positive if the state of the economy turns out to be good or a downturn. However, if it surprisingly turns out to be a recession then $\omega = r =>$ $(\pi_{w=r} - \pi_d) < 0$, and all assets are securitized so that $\frac{I_L^N}{S_L^N} = 1 < 1 + \pi_d - \pi$ then successful intermediaries go bankrupt. Unsuccessful intermediaries do not have high-quality investments, and thus receive payoff $(\pi_\omega - \pi_d)A * S_L^N$, which imply even worse outcome in a recession, break-even if a downturn and generate positive profits if good times.

Oppgave 2-b: Micro-effiency vs. macro-fragility tradeoff

Securitization, and the trading of securities, allows risk to be allocated to those who are willing to bear it. This also allows for better diversification, which reduces idiosyncratic portfolio risk (given that the securities are non-perfectly correlated). The mathematical proof of diversification was done by Markowitz, but for simplicity illustrated in this graph:



So in a microeconomic perspective, the financial markets is very good in terms of economic efficiency. With rational expectations, this also true from a macroeconomic perspective. But, as we will see in the next task, the problem of systemic risk arises when expectations are irrational. This causes a trade-off, and the Financial Supervisory Authority ("Finanstilsynet") has a central role of facilitating for transparency in the financial markets, regulating intermediaries with capital requirement, protecting non-professional investors, etc., in order both keep micro-efficiency high and macro-fragility low.

Oppgave 2-c: Neglected aggregate risk in GSV model

If the intermediaries have irrational expectations such risk has been irrationally neglected, this may have severe consequences. With securitization all financial intermediaries are equally diversified in the same securities, then every intermediary is exposed to each other's risk. If a recession occurs then, and securitization is not completed, only the successful intermediaries may survive. With full securitization however, all intermediaries go bankrupt when a recession hits in the model of GSV.

Oppgave 3-a: Rare disasters

The equity premium puzzle is about the premium equity investors receive above the risk-free rate. The baseline model by Mehra and Prescott using Lucas Tree for determining the equity premium is $r = \Upsilon\sigma^2$, e.g. risk aversion multiplied with risk (as variance). The model suggests that the risk premium should be about ~2% given variance observed and risk aversion suggested by behavioral finance research. However, empirical evidence shows that this equity premium is about ~6% over very long time series data. So, the model is not capturing the correct degree of risk aversion and/or risk, because investors require in fact a higher risk premium than determined by the baseline model. Barro's model focus on the risk, specifically risk for "rare disasters". At most of the time, GDP develops along its trend line with only minor deviations. However, sometimes there are major shocks from the far end of the distribution, which substantially reduce the output, and this is a rare disaster. Barro introduces a second term on the right-hand side for rare disasters, such that

$$r = \Upsilon\sigma^2 + pEb[(1-b)^{-\Upsilon}-1]$$

Where $p > 0$ is the probability of disaster and $0 \leq b \leq 1$ is the disaster size as measured by reduction in GDP. The term inside the brackets is the investors marginal utility of money during a crisis over a normal situation. You see that investors may risk losing their money when in fact they need them the most. Analysts often ignore this in their beta technology calculations of the equity's cost of capital because they exclude unnormal events from the dataset. Hence, the risk investors are facing is often underestimated according to Barro. Now, how do Barro derive his model?

Let x_t denote the growth rate in dividends (or consumption) at time t, and in normal times which occur with probability 1 – p the distribution of consumption is $\ln x_t = \ln g_t \sim N(\mu, \sigma^2)$. While in times of rare disasters, which occur with probability p, the distribution is $\ln x_t = \ln g_t + \ln(1-b) \sim N(\mu, \sigma^2)$. When $b > 0$ then $\ln(1 - b) < 0$, such that the growth rate is negative. As b approaches 1 the negative consequences become more severe. We assume crisis to be non-predictable and independent for g, such that b is a random variable. We can then define the unconditional expectation of g, which happens to equal the conditional expectation of x given normal times

$$Eg_t = E(X_t \mid \text{normal}) = e^{\mu + ½ \sigma 2}$$

The conditional expectations given a disaster may be written as …

$$E(X_t \mid \text{disaster}) = (1 - b)Eg_t - \text{cov}(b, g_t) = (1 - b)Eg_t$$

Such that we finally may define the unconditional expectation of x as

$$Ex_t = (1 - p)Eg_t + p(1 - Eb)Eg_t = (1 - pEb)Eg_t = (1 - pEb)e^{\{\mu + ½ \sigma 2\}}$$

Assuming power expected utility such that $\delta = \Upsilon$ and that disasters are independently and identically distributed, we know from Lucas Tree model that the expected return on equity is

$$Er_e = \frac{Ex}{\beta Ex^{1-\gamma}} = \frac{Ex}{\beta Ex^{1-\gamma}} = \frac{(1 - pEb)\exp^{\mu + ½ \sigma 2}}{\beta[1 - p + pE(1 - b)^{1-\gamma}]e^{(1-\gamma)\mu + (1-\gamma)^2 \frac{1}{2}\sigma^2}}$$

Developing into continuous time, the expected equity return can be approAnd the return rate on risk free asset is

$$r_f = \frac{1}{\beta Ex^{-\gamma}} = \frac{1}{\beta[1 - p + pE(1 - b)^{1-\gamma}]e^{-\gamma\mu + \frac{1}{2}\gamma^2\sigma^2}}$$

Developing expressions for expected equity return and risk-free return in continuous time it can be shown that the equity premium by Barro becomes

$$r = r_e - r_f = \Upsilon\sigma^2 + pEb[(1-b)^{-\Upsilon}-1]$$

So, if the probability of disaster is $p = 0$, then we're left with the baseline model. But as in Barro's model, the risk premium increases with the disaster probability p and expected disaster size b and the marginal utility of consumption during a disaster.

So, investors who take disasters into account, such as the corona virus etc., require a higher risk premium than those who not. However, it is not sure that this argument holds for real world examples. Recall from the GSV's model of task 2 that the financial crisis in 2008 was mainly a result of investors and the like ignoring low-probability risks. That's conflicting arguments ..

Oppgave 3-b: Habit formation

Meanwhile Barro focused on the risk parameter for explaining the observed risk premium, Campbell & Cochrane (C&C) focuses on the risk aversion parameter. The model of C&C take the notion that risk aversion is not only about preference orderings regarding the utility of current and future consumption. Risk aversion is also about the level of consumption the society gotten used, e.g. the habit. That is, the individual benchmark her consumption relative to the neighbours. C&C refer this to "Keeping up with the Jonesses".

So, we use the covariance expression for the equity premium and the Lucas Tree for this explanation. C&C replaces the preference labelling c_t for consumption with the difference $c_t - h_t$ where h_t is the level of habit which serves like "lower floor". So increased utility is only consumption in the excess of the habit level, and the households seek to at least satisfy their habit level from the past. As the level of consumption increase, the habit level may increase too (we'll come back to this point). If the change in consumption and habit are equal, the net gain in utility is zero.

I assume that gross log-normal growth rate in consumption is normally distributed

$$lnx_{t+1} = \mu + v_{t+1}, v_{t+1} \sim N(0, \sigma^2)$$

And that households exhibit habit-based Epstein-Zin preferences and maximize their value function

$$V_t(A_t) = \ max_{c_t, w_t} \left\{(1-\beta)(c_t - h_t)^{1-\delta} + \ \beta[E_t V_{t+1}(A_{t+1})^{1-\gamma}]^{\frac{1-\delta}{1-\gamma}}\right\}^{\frac{1}{1-\delta}}$$

Subject to the dynamic capital constraint $A_t = R_{t+1}(A_t - C_t) = [w_t R_{e,t+1} + (1-w_t)R_{f,t+1}](A_t - C_t)$

We further define a surplus consumption ratio $S_t = (C_t - h_t) / c_t$. If consumption equals the habit, the surplus ratio is zero – which isn't much good in terms of utility. If the ratio approaches one, the difference between consumption and habit is so large that habits is neglected. As mentioned above, an important feature is how the change in consumption and habits are related to each other throughout time. Campbell and Cochrane specify the relationship as

$$s_{t+1} = (1 - \emptyset)\bar{s} + \emptyset s_t + \lambda(s_t)(lnc_t - lnc_t - \mu)$$

This equation states that habit responds positively over time to changes in consumption in a non-linear fashion by $\lambda(s_t) > 0$.

When using our definition for the gross log-normal growth rate in consumption we can then write change in consumption surplus from time t to t + 1 as …

$$s_{t+1} - s_t = (\emptyset - 1)\bar{s} + \emptyset s_t + \lambda(s_t)v_{t+1}$$

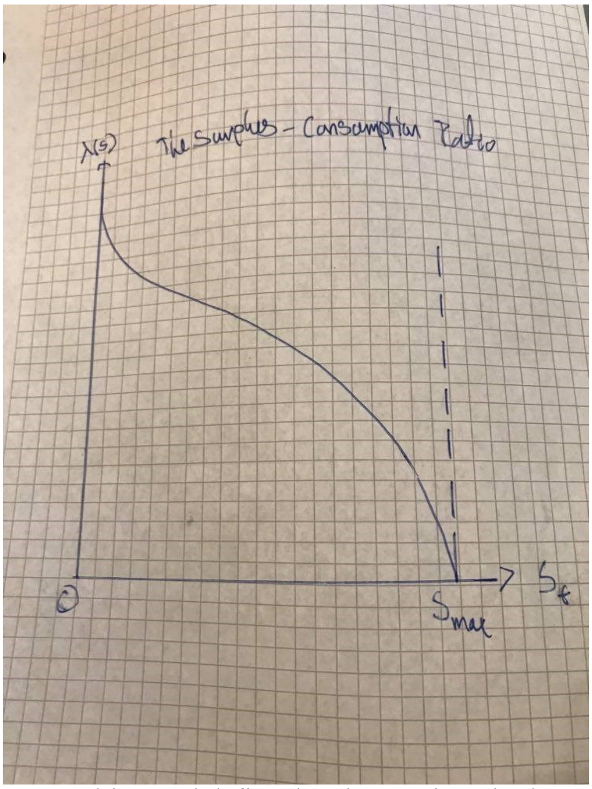
By using this result and some algebra we find that the stochastic discount factor on log form is

$$m_{t+1} = \frac{1-\gamma}{1-\delta}[ln\beta - \delta(\emptyset - 1)\bar{s}] - \gamma\mu - \frac{\delta - \gamma}{1-\delta} lnE_t(M_{t+1}x_{t+1}) - \left[\gamma + \delta\frac{1-\gamma}{1-\delta}\lambda(s_t)\right]\sigma^2$$

Which may be used to derive the risk premium for C&C 's habit formation model as

$$E_t r_{e,t+1} - r_{f,t+1} = -cov_t(r_{e,t+1}, m_{t+1}) = \left[\gamma + \delta\left(\frac{1-\gamma}{1-\delta}\right)E\lambda(s_t)\right]\sigma^2 \geq \gamma\sigma^2$$

Given Epstein-Zin preferences. This expression for the equity premium is equal or greater than the Lucas Tree version given that $\delta > 0$ and that $(1 - \Upsilon) / (1 - \delta) > 0$. This requirement is kind of odd though, but not too far stretched. But in the case if $\Upsilon > 1$, which it often is, then δ must be bigger than one as well (which by the way is the opposite of Bansal & Yaron's model who require $\delta < 0$). So, it's not as convincing as we'd like to. A nice thing about C&C's habit model of risk premium is that it varies with $\lambda(s)$ as depends on the consumption surplus ratio at time t.



The sketched graph of the surplus consumption ratio function shows that $\lambda(s)$ is always non-negative. In particular, $\lambda(s)$ is high when s_t is low. This means, when the consumption is close to the habit level, e.g. times are "bad", the marginal utility of increased consumption is high. This cause households to require a higher equity premium in order to take on extra risk during bad times. If the consumption is way above the habit level, such that we're at s_{max}, then $\lambda(s)$ is zero and the equity premium equation becomes $\Upsilon\sigma^2$ like in the Lucas Tree. No additional premium above the "normal" equity premium is required in such circumstances. This model fits the data quite nicely and some researchers consider this the end of the story of the puzzle.

If we move from Epstein-Zin preferences to power expected utility with constant relative risk aversion, the equity premium with habit become

$$E_t r_{e,t+1} - r_{f,t+1} = [1 + \mathrm{E}\lambda(s_t)]\gamma\sigma^2$$

Which is simpler and require no restrictions regarding δ and (1 – ϒ) / (1 – δ). However, a fair critique against C&C's model, is its dependency upon the specification of λ(s).