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1. Concepts ~~white~~

a) White noise

- A variable that is random with mean zero and constant variance. The white noise is said to be i.i.d. (independent, identically distributed), the variable is independent across time, independent: drawn from (*)

Let $\{\varepsilon_t\}$ be ~~the~~ white noise process.

We then have the following:

$$E(\varepsilon_t) = 0$$

$$E(\varepsilon_t^2) = var(\varepsilon_t) = \sigma^2$$

$$E(\varepsilon_t \cdot \varepsilon_{t-1}) = cov(\varepsilon_t, \varepsilon_{t-1}) = 0.$$

(*) different distributions at each point in time.

b) Weak stationarity.

→ Stationarity is a concept that states that the joint probability distribution ~~is not changed w~~ for a variable is not changed, when shifted in time.

For a variable, the criterias for weak stationarity is that the first and second moments are constant ((mean and variance, respectively):

For the $\{y_t\}$-series to be weakly stationary we have:

Mean $E(y_t) = \mu$

Variance $E[y_t \cdot y_t] = var(y_t) = \sigma^2$

With covariance

$E(y_t \cdot y_{t-s}) = cov(y_t, y_{t-s}) = \gamma_{t-s}$

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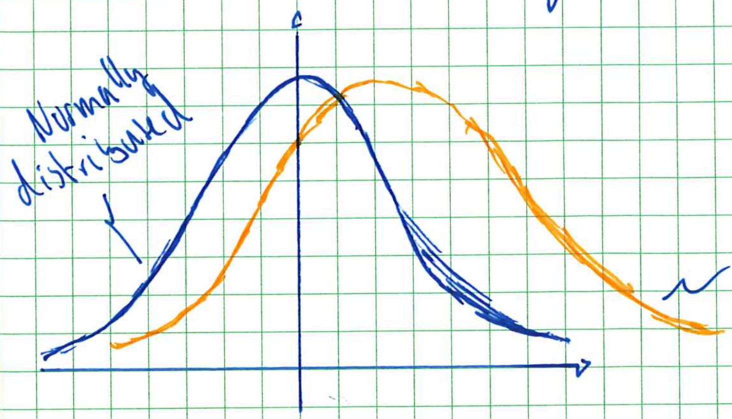
c) Skewness

For a given series, skewness is a measure of how much the probability distribution for the variable is skewed (no. "forskjøvet") to the left or right, relative to the normal distribution.

The skewness is given by the formula

$$\eta_3 = \frac{E[(y_t - E(y_t))^3]}{\sigma^3}, \quad \sigma = \sqrt{var(y_t)},$$

if we are looking at the series $\{y_t\}$.



Positive skewness.

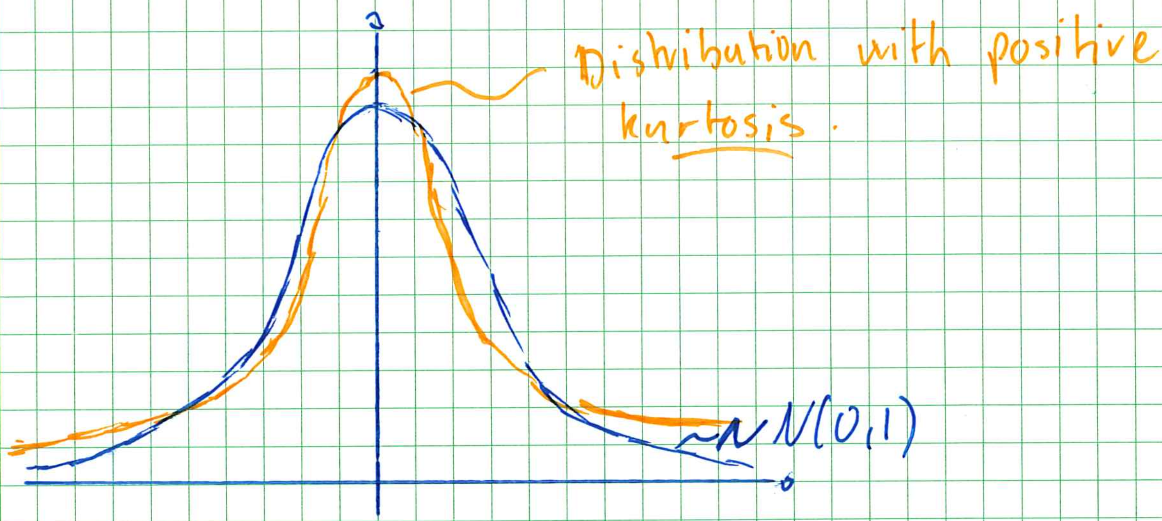
d) Kurtosis

Kurtosis measures the "fat-tailedness" of a probability distribution relative to the normal distribution.

Formula:

$$\eta_4 = \frac{E[(y_t - E(y_t))^4]}{\sigma^4}, \quad \sigma = \sqrt{var(y_t)}.$$

~~The~~ Students ~~t~~-distribution is an example of a distribution with fatter tails than the normal distribution. Illustrated:



Skewness and kurtosis are also called the third and fourth moments, respectively.

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2. Variable y_t is generated by

$$y_t = a_1 y_{t-1} + \varepsilon_t \quad , \quad \varepsilon_t \sim N(0, \sigma^2)$$

$$y_0 = 0$$

a) Complete solution $|a_1| < 1$.

$$y_t = a_1 y_{t-1} + \varepsilon_t$$

Insert for $y_{t-1} = a_1 y_{t-2} + \varepsilon_{t-1}$

$$y_t = a_1(a_1 y_{t-2} + \varepsilon_{t-1}) + \varepsilon_t$$

$$= a_1^2 y_{t-2} + a_1 \varepsilon_{t-1} + \varepsilon_t \quad , \quad \Big| \text{ insert for } y_{t-2}$$

$$= a_1^2 (a_1 y_{t-3} + \varepsilon_{t-2}) + a_1 \varepsilon_{t-1} + \varepsilon_t$$

$$= a_1^3 y_{t-3} + a_1^2 \varepsilon_{t-2} + a_1 \varepsilon_{t-1} + \varepsilon_t$$

$$\vdots$$

$$y_t = a_1^n y_{t-n} + \sum_{i=0}^{n} a_1^i \varepsilon_{t-i}$$

Solution when repeating the steps n times.

In the limit, we find,

$$\lim_{n \to \infty} y_t = 0 + \sum_{i=0}^{\infty} a_1^i \varepsilon_{t-i} \quad , \text{ as } |a_1| < 1$$

Hence, the solution to the difference equation is

$$y_t = \sum_{i=1}^{\infty} a_1^i \varepsilon_{t-i}$$

b) Expectation when $|a_1|<1$:

~~$E(y_t) = E[a_1 y_{t-1} + \varepsilon_t]$~~

~~$= 0$~~

~~Since~~

$$E(y_t) = 0 + E\left(\sum_{i=0}^{\infty} a_1^i \varepsilon_{t-i}\right)$$

$$\underline{E(y_t) = 0}$$

Since $\varepsilon_t \sim N(0, \sigma^2)$, and $y_0 = 0$

c) Variance : $var(y_t) = E[[y_t - Ey_t][y_t - Ey_t]] = E(y_t \cdot y_t)$

$$= E[(\varepsilon_t + a_1\varepsilon_{t-1} + a_1^2 \varepsilon_{t-2} + \cdots)(\varepsilon_t + a_1\varepsilon_{t-1} + a_1^2\varepsilon_{t-2} + \cdots)]$$

$$= E[\varepsilon_t^2 + a_1^2 \varepsilon_{t-1}^2 + a_1^4 \varepsilon_{t-2}^2 + \cdots + \text{cross products}]$$

Following the assumption that $\varepsilon_t \sim N(0, \sigma^2)$

$E[\text{cross products}] = 0$ and $E(\varepsilon_t^2) = E(\varepsilon_{t-1}^2) = \cdots = \sigma^2$

Hence

$$E(y_t \cdot y_t) = \sigma^2(1 + a_1^2 + a_1^4 + \cdots)$$

$$\Rightarrow \quad \underline{var(y_t) = \frac{\sigma^2}{1-a_1^2}}$$

d - Forecast of y_{t+2}, $|a_1|<1$

Update our original equation with one period:

$$y_{t+1} = a_1 y_t + \varepsilon_{t+1}$$

Take the conditional expectation:

$$E_t y_{t+1} = a_1 E_t y_t + E_t(\varepsilon_{t+1})$$

as a white noise-process is unforcastable

$E_t(\varepsilon_{t+1}) = 0$

Inserting for y_t:

$$E_t y_{t+1} = a_1 E_t[a_1 y_{t-1} + \varepsilon_t]$$
$$= a_1^2 y_{t-1} + a_1 \varepsilon_t$$

Update two periods (original equation)

$$y_{t+2} = a_1 y_{t+1} + \varepsilon_{t+2}$$

$$E_t y_{t+2} = a_1 E_t[y_{t+1}]$$
$$\cancel{= a_1 E_t[a_1^2 + a_1 \varepsilon_t]}$$
$$= a_1[a_1^2 y_{t-1} + a_1 \varepsilon_t]$$

$$E_t y_{t+2} = a_1^3 y_{t-1} + a_1^2 \varepsilon_t$$

Have to take with us the original white noise at ε_t since this is a "realized value" of the white noise process.

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e) Complete solution when $a_1 = 1$

Following the logic from the recursive solution of the difference equation i a) we find

$$y_t = a_1^n y_{t-n} + \sum_{i=1}^{n} a_1^i \varepsilon_{t-i}$$

~~In the limit we will no~~

Set $a_1 = 1$ and take the limit we find

$$y_t = y_{t-n} + \sum_{i=0}^{n} \varepsilon_{t-i}$$

$$\lim_{n\to\infty} y_t = y_0 + \sum_{i=0}^{\infty} \varepsilon_{t-i} \quad , \quad y_0 = 0$$

$$=0 \quad y_t = \sum_{i=0}^{\infty} \varepsilon_{t-i}$$

f. Expectation

$$E(y_t) = E\left(\sum_{i=0}^{\infty} \varepsilon_{t-i}\right) = E(\varepsilon_t + \varepsilon_{t-1} + \cdots)$$

as $\varepsilon_t \sim N(0,1)$

$E(y_t) = 0$, as we found earlier.

The variance however, will be changed:

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g.) Variance

$$var(y_t) = E[[y_t - Ey_t][y_t - Ey_t]] = E(y_t \cdot y_t)$$

$$var(y_t) = E[(\varepsilon_t + \varepsilon_{t-1} + \varepsilon_{t-2} + \cdots)(\varepsilon_t + \varepsilon_{t-1} + \varepsilon_{t-2} + \cdots)]$$

$$= E(\varepsilon_t^2 + \varepsilon_{t-1}^2 + \varepsilon_{t-2}^2 + \cdots + \text{cross products})$$

$E(\text{cross products}) = 0$, $E(\varepsilon_t^2) = E(\varepsilon_{t-1}^2) = \cdots = \sigma^2$

$$var(y_t) = \sigma^2 + \sigma^2 + \sigma^2 + \cdots$$

$$= \sum_{i=1}^{t} \sigma^2 = \underline{\underline{t\sigma^2}}$$

So the concept of weak stationarity, described in 1 b) does ~~not~~ ~~broken~~ not hold, as the variance will increase over time.

In the limit as $t \to \infty$ ~~...~~

$$\Rightarrow var(y_t) \to \infty$$

So the variance will explode, and the series will be non-stationary.

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h. Forecast when $a_1 = 1$

$$y_t = y_{t-1} + \varepsilon_t$$

$$y_{t+1} = y_t + \varepsilon_{t+1}$$

$$E_t\, y_{t+1} = y_t$$

$$y_{t+2} = y_{t+1} + \varepsilon_t$$

$$E_t\, y_{t+2} = E_t\, y_{t+1}$$

$$E_t\, y_{t+2} = y_t$$

i) Test for $a_1 = 1$

To test if $a_1 = 1$ one can use the Dickey-Fuller test, or the Augmented Dickey Fuller test.

~~With~~ base in the AR(1) model

$$y_t = a_1 y_{t-1} + \varepsilon_t$$

we rewrite this to yield

$$\Delta y_t = a_1 y_{t-1} - y_{t-1} + \varepsilon_t$$

$$\Delta y_t = (a_1 - 1) y_{t-1} + \varepsilon_t$$

$$\Delta y_t = \mu y_{t-1} + \varepsilon_t \quad , \quad \mu = a_1 - 1$$

If $a_1 = 1$ then $\mu = 0$ and we have what is called a unit root process. Our ~~test~~ hypothesis is

$$H_0: \mu = 0 \qquad (a_1 = 1)$$

$$H_A: \mu < 0 \qquad (a_1 < 1)$$

So it's a one sided test to see if ~~this~~ a_1 is statistically different from 1.

By rejecting H_0 we conclude that $a_1 < 1$.

If H_0 is not rejected we are dealing with a non-stationary series.

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To test the null against the alternative one ~~we~~ can not use classic inference methods, as y_t does not follow a t-distribution under the null. One can generate critical values by using Monte-Carlo methods, or ~~by~~ using the Dickey-Fuller critical values.

An extended version of the test, is the Augmented Dickey Fuller test, where longer lags on y_t is used. This is because ~~as~~ ε_t is assumed to be serially uncorrelated under the null. If this assumption is violated estimate instead.

$$\Delta y_t = \mu y_{t-1} + \sum_{i=2}^{p} \phi_i \Delta y_{t-i+1} + \varepsilon_t$$

$$\mu = -1 + \sum_{j=1}^{p} a_j \quad , \quad \phi_i = -\sum_{j=1}^{p} a_j \quad ,$$

with the underlying model

$$y_t = a_0 + a_1 y_{t-1} + a_2 y_{t-2} + \cdots + a_p y_{t-p} + \varepsilon_t$$

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3 IDENTIFYING THE PROCESS.

The process y_1 clearly has a decaying autocorrelation coefficient, the decay looks very much geometric. The partial autocorrelation has a single spike at lag 1, and some small insignificant spikes later on.

I believe y_1 is an AR(1)-process given by:

$$y_{1t} = a_1 y_{1t-1} + \varepsilon_t \quad , \text{with } 0<a_1<1$$

To justify this, let's have a look at the variance and autocovariances

VARIANCE: $E[y_t - Ey_t][y_t - Ey_t] = E(y_t \cdot y_t)$

$$E[(\varepsilon_t + a_1\varepsilon_{t-1} + a_1^2\varepsilon_{t-2} + \cdots)(\varepsilon_t + a_1\varepsilon_{t-1} + a_1^2\varepsilon_{t-2} + \cdots)]$$

$$= E[\varepsilon_t^2 + a_1^2\varepsilon_{t-1}^2 + a_1^4\varepsilon_{t-2} + \cdots + \text{cross products}]$$

Assuming ε_t is white noise $\varepsilon_t \sim N(0,\sigma^2)$ we find that $E(\varepsilon_t^2) = E(\varepsilon_{t-1}^2) = \cdots = \sigma^2$ and $E[\text{cross products}] = 0$

Thus $\operatorname{var}(y_t) = \sigma^2(1 + a_1^2 + a_1^4 + a_1^6 + \cdots)$

$$var(y_t) = \frac{\sigma^2}{1-a_1^2}$$

Autocovariances:

$$E(y_t - E(y_t))(y_{t-1} - E(y_{t-1})) = E(y_t \cdot y_{t-1})$$

$$= E[(\varepsilon_t + a_1\varepsilon_{t-1} + a_1^2\varepsilon_{t-2} + \cdots)(\varepsilon_{t-1} + a_1\varepsilon_{t-2} + a_1^2\varepsilon_{t-3} + \cdots)]$$

$$= a_1\sigma^2(1 + a_1^2 + a_1^4 + \cdots)$$

$$\gamma_1 = \frac{a_1\sigma^2}{1-a_1^2}$$

Continuing in this fashion, we find that

$$\gamma_s = \frac{a_1^s\sigma^2}{1-a_1^2} \quad \forall\, s \geq 1$$

The autocorrelations is defined as

$$\rho_s = \frac{\gamma_s}{\gamma_0}, \text{ where } \gamma_0 = var(y_t)$$

So we find that

$$\rho_1 = \frac{\gamma_1}{\gamma_0} = \frac{\frac{a_1\sigma^2}{1-a_1^2}}{\frac{\sigma^2}{1-a_1^2}} = a_1 \quad \text{, and}$$

$$\rho_2 = \frac{\gamma_2}{\gamma_0} = \frac{\frac{a_1^2\sigma^2}{1-a_1^2}}{\frac{\sigma^2}{1-a_1^2}} = a_1^2$$

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So $\rho_s = a_1^s$

With $0<a_1<1$ we see that the autocorrelation $\rho_s = a_1^s$ will give a ~~dec~~ geometrically decaying autocorrelation function.

The partial autocorrelations are given

$$\phi_{11} = \rho_1$$

$$\phi_{22} = \frac{cov(y_t, y_{t-2} | y_{t-1})}{\sqrt{var(y_t | y_{t-1}) \cdot var(y_{t-2} | y_{t-1})}}$$

Or can be found through the regression

$$y_1 = \phi_{11} y_{t-1} + \varepsilon_t$$

$$y_2 = a_1 y_{t-1} + \phi_{22} y_{t-1} + \varepsilon_t$$

$$\vdots$$

$$y_t = a_1 y_{t-1} + a_2 y_{t-2} + \dots + \phi_{ss} y_{t-s} + \varepsilon_t$$

As the first partial autocorrelation for y_1 is equal to ρ_1 and the rest is zero, I conclude that y_1 is following an AR(1) process.

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y_2-series

For the y_2-series, we need to find a process where the ~~AC~~ autocorrelation ~~becomes~~ becomes zero ~~for~~ at the second lag, ~~that is~~ we need

$$\gamma_2 = E[y_t - Ey_t][y_{t-2} - Ey_{t-2}] = 0$$

By looking at the autocovariances for an MA(1) process we find.

$$y_t = \varepsilon_t + \beta_1 \varepsilon_t$$

Variance: $\gamma_0 = E(y_t - Ey_t)(y_t - Ey_t)$

$$= E[(\varepsilon_t + \beta_1 \varepsilon_{t-1})(\varepsilon_t + \beta_1 \varepsilon_{t-1})]$$

$$= \underline{\sigma^2(1+\beta_1^2)}$$

1. autocov. $\gamma_1 = E[(\varepsilon_t + \beta_1 \varepsilon_{t-1})(\varepsilon_{t-1} + \beta_1 \varepsilon_{t-2})]$

$$\underline{\gamma_1 = \beta_1 \sigma^2}$$

2. Autocov. $\gamma_2 = E[(\varepsilon_{t-2} + \beta_1 \varepsilon_{t-3})(\varepsilon_t + \beta_1 \varepsilon_{t-1})]$

$$\underline{\gamma_2 = 0}$$

Hence

$$\rho_1 = \frac{\gamma_1}{\gamma_0} = \frac{\beta_1 \sigma^2}{(1+\beta_1^2)\sigma^2} = \frac{\beta_1}{1+\beta_1^2}$$

$$\rho_2 = 0$$

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This tells me that I'm looking for an MA(1) process. If only I had the time to justify that using the PACF.

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Q 4

$$\begin{pmatrix} y_{1t} \\ y_{2t} \end{pmatrix} = \begin{pmatrix} a_{10} \\ a_{20} \end{pmatrix} + \begin{pmatrix} a_{11,1} & a_{12,1} \\ a_{21,1} & a_{22,1} \end{pmatrix} \begin{pmatrix} y_{1,t-1} \\ y_{2,t-1} \end{pmatrix}$$

$$+ \begin{pmatrix} a_{11,2} & a_{12,2} \\ a_{21,2} & a_{22,2} \end{pmatrix} \begin{pmatrix} y_{1,t-2} \\ y_{2,t-2} \end{pmatrix} + \begin{pmatrix} e_{1t} \\ e_{2t} \end{pmatrix}$$

By writing the system on general form (that means doing the matrix multiplication) we find:

$$y_{1t} = a_{10} + a_{11,1} y_{1,t-1} + a_{12,1} y_{2,t-1} + a_{11,2} y_{1,t-2} + a_{12,2} y_{2,t-2} + e_{1t} \qquad (1)$$

$$y_{2t} = a_{20} + a_{21,1} y_{1,t-1} + a_{22,} y_{2,t-1} + a_{21,2} y_{1,t-2} + a_{22,2} y_{2,t-2} + e_{2t} \qquad (2)$$

For y_2 to cause y_1 we need to find that lags of y_2 help explain y_1.

Hence, we can use an F-test to test for joint significance of the variables $a_{12,1}$ and $a_{12,2}$ in equation (1)

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Our hyphothesis will be formulated as

$H_0: a_{12,1} = a_{12,2} = 0$

$H_A: a_{12,1} \neq 0 \vee a_{12,2} \neq 0$ (jointly $\neq 0$)

For us to conclude that y_2 causes y_1 we would like to reject the null with clear margin at conventional levels.

If we are talking about Granger cause, then lags of y_1 should not help explain y_2, hence we need to test:

$H_0: a_{21,1} = a_{21,2} = 0$, against

$H_A: a_{21,1} \neq 0 \vee a_{21,2} \neq 0$ (jointly $\neq 0$)

in equation (2) and keep the null hypothesis with good margin at conventional levels.

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5 Information Criteria

The information criterias are defined as

$$AIC = \ln(\tilde{\sigma}^2) + \frac{2k}{T}$$

$$SBC = \ln(\tilde{\sigma}^2) + \frac{k \cdot \ln(T)}{T}$$

$k = p+q+1$, and is the number of paramaters. $\tilde{\sigma}^2$: ~~sum of~~ square of estimated residuals

The difference between the two lies in the second term on the right hand side.

As $\ln(T) > 2$ for $T > 8$, and we normally use sample sizes bigger than 8, SBC penalize the inclusion of more variables harder than AIC. SBC will in general pick ~~the~~ a more parsimonous model than AIC because of this penalty.

One would like to pick the model with the lowest AIC and SBC, since that model

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gives a good fit due to the low value of $\tilde{\sigma}^2$.

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For general interpretation

6. The models are obviously different kinds of GARCH specifications, with an AR(2) model in the mean equation. The models can be formulated

$$y_t = a_0 + a_1 y_{t-1} + a_2 y_{t-2} + \varepsilon_t$$

$$\varepsilon_t = v_t \sqrt{h_t}$$

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}$$

~~We are interested in~~

We want to check that the estimated parameters are statistically significant, and to check if the stability condition: $\alpha_0 > 0$ and $0 < \alpha_1 + \beta_1 < 1$ holds.

a) M1

Regular GARCH specification. Have the following t-values for the different parameters:

t_{a_0}	$= -13{,}69$	t_{α_0}	$= 10{,}735$
t_{a_1}	$= -3{,}08$	t_{α_1}	$= 19{,}1512$
t_{a_2}	$= -9{,}07$	t_{β_1}	$= 55{,}75$

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All the t-values are significant at the 5% level (critical value ±1,96).

Find:

$\alpha_0 = 0,0433160 > 0$

and $\alpha_1 + \beta_1 = 0,892256 < 1$

So the stability condition holds.

b. M2

t-values

$t_{a_0} = 4,4201$	$t_{d_0} = 3,2200$
$t_{a_1} = -3,5974$	$t_{a_1} = 8,5793$
$t_{a_2} = -2,56547$	$t_{\beta_1} = 94,1573$

Find that all parameters are significant at the 5%-level.

Stability condition

$\alpha_0 = 0,00925802 > 0$

$\alpha_1 + \beta_1 = 0,9918103 < 1$ (marginally).

c) M3

Observe that $\alpha_1 + \beta_1 = 0{,}090399 + 0{,}909601$
$= 1$

Hence, we have an IGARCH model, or integrated GARCH.

We can thus write the conditional variance

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}$$

~~$h_t = \alpha_0 + (1-\beta_1)$~~

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + (1-\alpha_1) h_{t-1}$$

as $\alpha_1 + \beta_1 = 1$

An IGARCH model show a high degree of persistence, and the forecast of the conditional variance ~~will~~ not converge ~~under~~ ~~the~~ on the unconditioanl variance

$$var(\varepsilon_t) = \frac{\alpha_0}{1-\alpha_1-\beta_1}.$$

Significance of the parameters:

$t_{a_0} = 4{,}24784$	$t_{\alpha_0} = 3{,}4633$
$t_{a_1} = -3{,}9603$	$t_{\alpha_1} = 9{,}5054$
$t_{a_2} = -2{,}5172$	$t_{\beta_1} = $ —

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All the estimated parameters are significant at the 5% -level. Also $\alpha_0 = 0{,}008658 > 0$.

d) M4

The underlying model is now a Threshold GARCH (TGARCH) model. Also called GJR after its inventors; Glosten, Jagannathan and Runkle.

The model is used when one believe that there is asymmetric effect of ~~[illegible]~~ shocks, so that "bad news" may have a greater effect on volatility than good news.

Formulation of conditional volatility:

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1} + \lambda_1 \varepsilon_{t-1}^2 \cdot I_{t-1}$$

$$I_{t-1} = \begin{cases} 1 & \text{if } \varepsilon_{t-1} < 0 \\ 0 & \text{otherwise} \end{cases}$$

Observe that we get an extra effect on the conditional volatility if $\varepsilon_{t-1} < 0$:

Effect of "Bad news": $\alpha_1 + \lambda_1$

Effect of "good news": α_1

t-values

$t_{a_0} = 2{,}4939$
$t_{a_1} = -3{,}82076$
$t_{a_2} = -2{,}1739$

$t_{a_0} = 4{,}0384$
$t_{a_1} = -3{,}1862$
$t_{\beta_1} = 105{,}661$
$t_{\lambda_1} = 8{,}5064$

All the estimated parameters are significant at the 5% level!

Stability conditions

$\alpha_0 = 0{,}009784 > 0$

$\alpha_1 + \beta_1 = 0{,}132293 < 1$

e) M5. The model of the conditional variance is now and EGARCH model (E for exponential). This model also allows for asymmetric effects of volatility, observe this by using the underlying model for the conditional variance:

$$\ln h_t = \alpha_0 + \alpha_1\left(\frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}}\right) + \lambda_1\left|\frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}}\right| + \beta_1 \ln h_t$$

"Good news" ($\varepsilon_{t-1} > 0$) are expected to have the effect $\alpha_1 + \lambda_1$, while "bad news" ($\varepsilon_{t-1} < 0$) are expected to have the effect
$-\alpha_1 + \lambda_1$

Significance of parameters:

$t_{a_0} = 2,1402$

$t_{a_1} = -3,7577$

$t_{a_2} = -1,7081$

$t_{\alpha_0} = -1,1956$

$t_{a_1} = -9,2486$

$t_{\lambda_1} = 7,952$

$t_{\beta_1} = 322,93$

Now the coefficient on y_{t-2} (a_2) and the coefficient ~~on~~ α_0 (intercept in conditional variance) are insignificant.

Also the intercept is negative:

$\alpha_0 < 0 \quad , \alpha_0 = -0,328754$

~~which~~ This ~~which~~ implies a negative unconditional variance, which is not possible.

The stability condition

$0 < \alpha_1 + \beta_1 < 1 \quad : \quad \alpha_1 + \beta_1 = 0,857543$

is fulfilled.

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f) What model to choose:

Model	Log-Likelihood	AIC	SBC
M1	-4641,6886	2,8444	2,8855
M2	-4574,3674	2,8038	2,8168
M3	-4576,517	2,8028	2,8139
M4	-4519,885	2,7693	2,7843
M5	-4522,8942	2,7712	2,7861

I use the model selection criteria AIC and SBC when I chose model. Theese are defined in question 5, but as we ~~use~~ estimate GARCH models, the estimation is done by using maximum likelihood, hence the term $\ln(\hat{\sigma}^2)$ in AIC & SBC is now changed to $\left(-\frac{2\ln L}{T}\right)$.

From the table above observe that both AIC and SBC prefer the TGARCH model M4. Hence I choose this model. All the estimated parameters are significant at the 5%-level and the stability conditions are fullfilled for M4, so there should be no trouble choosing this.

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6. Interpretation of results.

M1: Lags of y_t have negative effect on y_t and an increase in y_{t-1} with one unit leads to a decrease in y_t with 0,057 units. An increase in y_{t-2} with one unit, decreases y_t with 0,03828 units.

- ~~Var~~ h_t ÷ function:

Last periods conditional volatility (h_{t-1})

One unit increase gives 0,790 units increase in h_t. For the residuals ε_{t-1}^2 ~~the~~ a one unit increase yields ~~an~~ increase with 0,1022 units ~~in~~ in h_t.

M2: Conditional variance more dependent ~~on~~ ~~last~~ last periods realization $\hat{h}_{t-1}$ leads to higher persistence in the conditional variance and shocks will last longer. ~~I am not able to~~

Based on the underlying model (GARCH) I think this result is a ~~bit~~ weird compared to ~~the~~ ~~first~~ M1 which also is ~~an~~ GARCH specification.

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M3 : IGARCH, as described earlier.

M4 : Threshold GARCH model. Observe that "bad news" ~~or~~ a negative $\hat{\varepsilon}_{t-1}$ yields an extra effect on the conditional volatility with 0,132293 units. "Good news", or a positive ε_{t-1}, decrease conditional volatility by 0,021886 units.

M5 : EGARCH. The conditional volatility is affected negatively by the presence of the term

$\alpha_1\left(\frac{\hat{\varepsilon}_{t-1}}{\sqrt{\hat{h}_{t-1}}}\right)$. This there to capture the effects of a negative a ε_{t-1}.

If $\varepsilon_{t-1} > 0$ we get the effect

$\alpha_1 + \lambda_1$

If $\varepsilon_{t-1} < 0$ we get the effect $-\alpha_1 + \lambda_1$, hence the model suggests that $\varepsilon_{t-1} < 0$.

Underlying model for h_t:

$$\ln h_t = \alpha_0 + \alpha_1\left(\frac{\hat{\varepsilon}_{t-1}}{\sqrt{\hat{h}_{t-1}}}\right) + \lambda_1\left|\frac{\hat{\varepsilon}_{t-1}}{\sqrt{\hat{h}_{t-1}}}\right| + \beta_1 \ln h_{t-1}$$

7. A STAR (Smooth transitional Auto-regressive) model is a class of models specifications used in the field of regime shifting models.

Generally one would like to use a regime shifting model when the series switches ~~from~~ between different states ("regimes").

If the states or regimes are observable we can use regime shifting models that depend upon a observable shifting variable, s_{t-k}.

(If the states are unobservable, one often use Markov switching models).

To get a better grasp of STAR-models, i first explain a TAR model.

A TAR (Threshold AutoRegressive) model, is a model that can be formulated as

$$y_t = \begin{cases} \mu_1 + \phi_1 y_{t-1} + e_{1t} & \text{if } s_{t-k} < r \\ \mu_2 + \phi_2 y_{t-1} + e_{2t} & \text{if } s_{t-k} \geq r \end{cases}$$

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This is a model with two states or regimes. s_{t-k} is the state variable and defines when the model shifts from one regime to another, the shift is decided by the threshold value r.

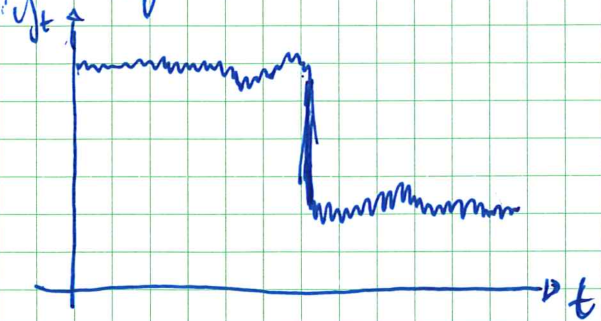
Using dummy-variables we can write:

$$y_t = [\mu_1 + \phi_1 y_{t-1} + e_{1t}][1 - D(s_{t-k} > r)]$$
$$+ [\mu_2 + \phi_2 y_{t-2} + e_{2t}][D(s_{t-k} > r)],$$

where $D(s_{t-k} > r) = 1$ if $s_{t-k} > r$, and zero otherwise.

Observe that the model will make an abrubt jump between the two states.

Graphically:



This formulation might not be feasible as the transition between two states ~~are~~ not

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may be better explained by a ~~transi~~ smoother transition between the two regimes.

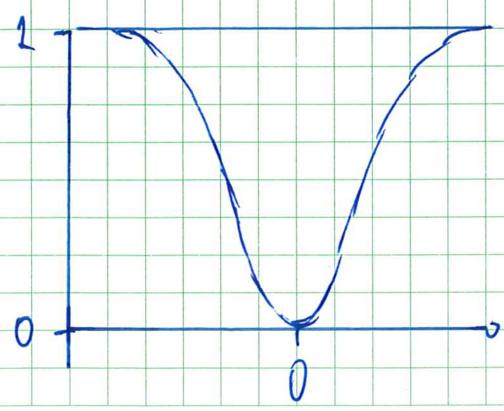
Using STAR models we specify a continous function that changes continously from zero to one, which allows for a smooth transition between the states.

Two much used specifications are the ESTAR (exponential STAR) and LSTAR (logistic STAR) specifications.

The ESTAR specification, is given as

$$G(y_{t-1};\gamma,r) = 1 - \exp(-\gamma(y_{t-1}-r)^2),$$

and the transition function can be represented graphically like



The nice thing about the function is that the transition will be equal from above or below, as

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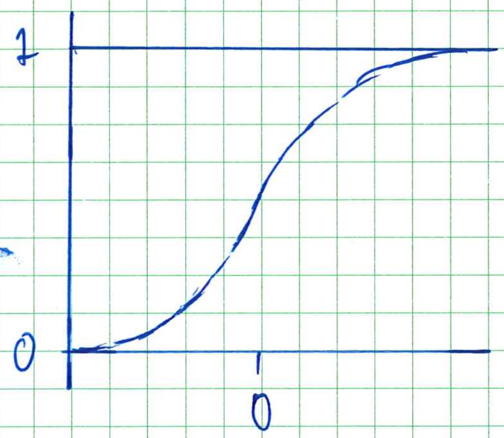
$$\lim_{y_{t-1} \to \pm\infty} = 1 \; .$$

The LSTAR specification is given as

$$G(y_{t-1} : \gamma, r) = \frac{1}{1 + \exp(-\gamma(y_{t-1} - r))} \; ,$$

and can be described graphically:



For both functions γ represents the speed of transition (between the to states). For the LSTAR

$$\gamma \to \infty \Rightarrow G(y_{t-1} : \gamma, r) = 1 \; ,$$

and we have a general TAR.

By now ~~using~~ writing the STAR model as:

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$$y_t = [\mu_1 + \phi_1 y_{t-1} + e_{1t}][1 - G(y_{t-1}; \gamma, r)]$$

$$+ [\mu_2 + \phi_2 y_{t-1} + e_{2t}][G(y_{t-1}; \gamma, r)]$$

One can see that the STAR specification yields a smooth transition between the states.