FIN3006 – Applied Time Series Econometrics
Candidate number 1004
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Question 1

<u>Problem a</u>)

"Traditional "rockets and feathers" literature builds on the assumption that gasoline and crude oil prices are cointegrated. Define cointegration and explain why it is a useful framework for the gasoline and crude oil prices behavior."

Let x_t be a $k\ x\ 1$ vector of variables, then the elements of x_t are **cointegrated of order** $(\boldsymbol{d}, \boldsymbol{b})$ if

- All elements in x_t are $I(d), d > 0$
- There exists at least one vector of coefficients α such that $\alpha' x_t \sim I(d-b)$ where $b > 0$

As an example: Consider the processes oil_t, $gasoline_t \sim I(1)$. If we find a linear combination of $oil_t, gasoline_t$ such that this combination $\beta_1 oil_t + \beta_2 gasoline_t \sim I(0)$, then these two time series processes are cointegrated of order $(1,1)$. Formally,

$$oil_t, gasoline_t \sim CI(1,1)$$

If this holds, the vector (β_1, β_2) is the *cointegrating vector*. For example, if we define

$$z_t = oil_t - \alpha gasoline_t$$

Where $z_t \sim I(0)$ and $oil_t, gasoline_t \sim I(1)$, then the cointegrating vector is $(1, -\alpha)$. The variable z_t is in this case called the *disequilibrium* because it captures the difference between oil_t and $gasoline_t$ from their long-run equilibrium in which $oil_t = \alpha gasoline_t$.

Intuitively, it is logical to at least consider the possibility of cointegration between the two variables. Since oil is an important input in the production of gasoline, clearly the price of oil will have some impact on the price of gasoline from a microeconomic perspective. A cointegration analysis can then be used to predict future gasoline prices as a function of future oil prices (or vice versa) as it is just a simple linear combination of the two variables. To give one example of how this is useful: If the two processes are cointegrated and you (as an oil analyst for instance) observe a large gap between oil and gasoline prices *today*, then you can make the claim that you expect this gap to narrow down over time because these variables are bound to reach some long-run equilibrium as a result of the cointegration between them. Then, depending on the estimates, you can also perhaps make a claim of which variable will move up and down in response to the other by looking at the coefficient estimates.

Problem b)

"Consider now the analysis performed on line 35 and 43 of the do-file and line 23 and 31 of the log-file. Interpret the coefficients and compare the results between the two countries"

The estimate of the first model is

$$\begin{aligned}\Delta \ln us_t = 0 &+ 0.364\Delta \ln us_{t-1} + 0.157\Delta \ln us_{t-2} \\ &+ 0.0628\Delta \ln us_{t-3} \\ &+ 0.123\Delta \ln wti_{t-1} - 0.0425\Delta \ln wti_{t-2} + 0.0166\Delta \ln wti_{t-3} - 0.036\hat{\epsilon}_{t-1}\end{aligned}$$

Where us_t is the gasoline price in the U.S. Since $\Delta \ln us_t = \ln us_t - \ln us_{t-1}$, a unit change in this variable can be interpreted as a percentage change in the gasoline price. Then, the interpretation becomes

- If the gasoline price has been zero for the last three years, and the WTI crude oil price has been zero for the last three years, the expected price change in the gasoline price today is approximately zero
  - Not significant at even $\alpha = 10\%$
- If the total increase in the gasoline price from $t-2$ to $t-1$ is 1%, then the expected increase in gasoline prices from $t-1$ to t is 0.364%
  - Significant at $\alpha = 1\%$
- If the total increase in the gasoline price from $t-3$ to $t-2$ is 1%, then the expected increase in gasoline prices from $t-1$ to t is 0.157%
  - Significant at $\alpha = 1\%$
- If the total increase in the gasoline price from $t-4$ to $t-3$ is 1%, then the expected increase in gasoline prices from $t-1$ to t is 0.628%
  - Significant at $\alpha = 10\%$
- If the total increase in the WTI crude oil price from $t-2$ to $t-1$ is 1%, then the expected increase in gasoline prices from $t-1$ to t is 0.123%
  - Significant at $\alpha = 1\%$

- If the total increase in the WTI crude oil price from $t-3$ to $t-2$ is 1%, then the expected increase in gasoline prices from $t-1$ to t is -0.0425%
    - Significant at $\alpha = 1\%$
- If the total increase in the WTI crude oil price from $t-4$ to $t-3$ is 1%, then the expected increase in gasoline prices from $t-1$ to t is 0.0166%
    - Not even significant at $\alpha = 10\%$
- If the error in $t-1$ is 1%, then the expected increase in gasoline prices from $t-1$ to t is -0.036%
    - Significant at $\alpha = 1\%$

The estimate for the second model is

$$\begin{aligned}\Delta \ln ger_t = 0 - 0.188\Delta \ln ger_{t-1} - 0.005\Delta \ln ger_{t-2} \\ - 0.0123\Delta \ln ger_{t-3} \\ + 0.411\Delta \ln brent_{t-1} + 0.0149\Delta \ln brent_{t-2} + 0.0933\Delta \ln brent_{t-3} - 0.052\hat{v}_{t-1}\end{aligned}$$

The interpretation is similar as in the first model, but now for Germany and using Brent crude oil instead of WTI crude oil

- If the gasoline price in Germany has been zero for the last three years, and the Brent crude oil price has been zero for the last three years, the expected price change in the gasoline price today is approximately zero
    - Not significant at even $\alpha = 10\%$
- If the total increase in the gasoline price from $t-2$ to $t-1$ is 1%, then the expected increase in gasoline prices from $t-1$ to t is -0.188%
    - Significant at $\alpha = 1\%$
- If the total increase in the gasoline price from $t-3$ to $t-2$ is 1%, then the expected increase in gasoline prices from $t-1$ to t is -0.005%
    - Not significant at even $\alpha = 10\%$
- If the total increase in the gasoline price from $t-4$ to $t-3$ is 1%, then the expected increase in gasoline prices from $t-1$ to t is -0.012%
    - Significant at $\alpha = 10\%$

- If the total increase in the Brent crude oil price from $t-2$ to $t-1$ is 1%, then the expected increase in gasoline prices from $t-1$ to t is 0.41%
    - Significant at $\alpha = 1\%$
- If the total increase in the Brent crude oil price from $t-3$ to $t-2$ is 1%, then the expected increase in gasoline prices from $t-1$ to t is 0.0149%
    - Not significant at even $\alpha = 10\%$
- If the total increase in the Brent crude oil price from $t-4$ to $t-3$ is 1%, then the expected increase in gasoline prices from $t-1$ to t is 0.093%
    - Significant $\alpha = 5\%$
- If the error in $t-1$ is 1%, then the expected increase in gasoline prices from $t-1$ to t is -0.052%
    - Significant at $\alpha = 1\%$

So, now we can point out some general observations of the estimates.

The U.S gasoline price seems to generally increase if it has increased in the periods before. This result seems to be strong as the regression coefficients are significant, except for the $t-3$ term where it is significant at $\alpha = 6\%$ but not 5%. However, it does so in a decaying manner. In other words, price jumps in the past seem to have a decaying effect on future gasoline prices as a function of the time difference. This observation does not hold in Germany. In Germany, this effect is negative. However, this effect is only significant in the short run (last period).

With regards to the effect of the oil price, we see that the WTI crude oil has alternating effects on the gasoline price. The first lagged oil price increase raises the gasoline price, while the second lagged oil price has a negative increase on the gasoline price. The third is insignificant. This is somewhat counterintuitive as microeconomic theory suggests that prices should increase when the prices of inputs increase. Anyway, one explanation could be that the price increase is already "absorbed" in the periods before. In Germany, we find strictly positive results although two of the lagged variables are insignificant. This makes it hard to compare the overall effect with the similar variable in the U.S.

Finally, the error correction term captures by the lagged residual attempts to explain how the gasoline price is expected to change as a function of the disequilibrium of the last period. In both Germany and the U.S, a positive disequilibrium (oil price larger than what is predicted by the model), more formally

$$oil_t - \widehat{oil}_t = \hat{\epsilon}_t$$

Is met with a price fall in the gasoline price. Therefore, if the disequilibrium is positive, it seems like the gasoline price corrects itself toward the long-run equilibrium value by falling. In Germany, this equilibrium is met by a fall in -0.05% while in the U.S it is -0.036%, both highly significant. These are the *speed of adjustment* coefficients and from the value of these we can see that the effects are quite similar in magnitude.

The overall conclusion is that that the U.S gasoline prices responds somewhat differently from its past realizations than the gasoline price in Germany. Beyond this, the behavior is more or less the same, especially with regard to the error correction of a disequilibrium.

Problem c)

"Compare the ECM strategy with the Johansen methodology, emphasizing why the former might be appropriate in this context."

Before we compare the strategies, I find it appropriate to outline the Johansen methodology first. The Johansen methodology is a multivariate method that can allow us to determine *multiple* cointegrating relationships. This is done by performing a multivariate Dickey-Fuller test by estimating a vector autoregression model (VAR(1) for instance)

$$X_t = \alpha + BX_{t-1} + \epsilon_t$$

Where X_t is a $n\ x\ 1$ vector of integrated variables, say $\ln us_t$ and $\ln ger_t$. Before we proceed, the variables must be integrated of the same order. We reparametrize the model by taking the first difference

$$\Delta X_t = \alpha + (B - I)X_{t-1} + \epsilon_t$$

$$\Delta X_t = \alpha + \Pi X_{t-1} + \epsilon_t$$

For the same reason as in the univariate augmented Dickey-Fuller test, we may have to extend the differenced VAR(1) to remove autocorrelation in the error term. In general, we might need a model such as

$$\Delta X_t = \alpha + \Pi X_{t-1} + \sum_{i=1}^{q} \Gamma_{t-i} \Delta X_{t-i} + \epsilon_t$$

Then the common procedure is to estimate the VAR of the *undifferenced* data. The Johansen method is sensitive to the lag length, so the lag length of the VAR must be carefully selected by using information criteria for instance.

If X_t contains $I(1)$ processes, then $\Delta X_t \sim I(0)\ \forall\ t$. If ϵ_t is white noise, then the matrix ΠX_{t-1} must be stationary as the sum of stationary variables is itself stationary. The rank of the matrix Π will give us the amount of linearly independent rows in the matrix, should ΠX_{t-1} be stationary. This will again determine how many cointegrating relationships we have because this linear combination is used on a set of $I(1)$ variables that has now become $I(0)$.

In general,

- $rank(\Pi) = 0$: All variables are integrated, and we do not have any cointegrating relationship
- $rank(\Pi) = r < n$: We have r cointegrating relationships
- $rank(\Pi)$ = n: All variables are stationary, then we cannot have any cointegrating relationships

We determine the amount of relationships and the cointegrating vectors by using either a trace or max test.

The ECM strategy is more appropriate because we are simply looking at only two relationships, so there can only be one cointegrating relationship. The Johansen method is more appropriate when we want to look at $n > 3$ variables because then we can determine *multiple* cointegrating relationships. One problem that often arises we attempt to perform a cointegration analysis using the standard Engle-Granger method is that the choice of dependent variable matters if there are more than two, which the Johansen method avoids. In the two-equation case, this is of no concern and especially since the direction of causality is "obvious" in the sense that we should believe that oil prices affect gasoline prices and not the other way around. Finally, the first strategy will tend to deliver estimates with lower variance which are consistent in large samples by construction of the method of OLS. Of course, the importance of this final point depends on what the intention of the analysis is, but it should be pointed out.

Problem d)

"Discuss limitations to the analysis presented in the exercise (excluding what you have already discussed in the previous point). If you had access to the dataset, is there anything you would have done differently?"

Less verbose and detailed answer

- Account for break and use ADF instead of normal DF to check the order of integration on the variables
  - This can be done using a regime switching model such as a TAR
- If they are integrated of the same order, perform Engle-Granger procedure, but this time test residuals using an appropriate ADF and test whether the ADF specification is correct
- Cheat by restricting the period before/after the break and perform the cointegration analysis
  - After the break may give misleading results due to low sample size
  - Before the break will probably yield more convincing results as the sample size is larger

More verbose and detailed answer

When we look at the graphs of oil- and gasoline prices, there are two things that can be noted

- The price has increased over time which indicates non-stationarity
- There is a clear break in around 2008, and the behavior pre-2008 and post-2008 seems to be different

Non-stationarity will of course complicate our analysis because the time series process becomes harder to forecast since its behavior is constantly changing. In addition, we cannot perform any inference tests because they require stationarity. Therefore, we should test for it using a Dickey-Fuller test. So, let us outline the test before we proceed.

Consider the simple AR(1)

$$y_t = \phi y_{t-1} + u_t$$

Here u_t is **assumed and required** to be a white noise process. We wish to test whether $\phi = 1$ against $\phi < 1$. Thus, the formal hypotheses are

$$H_0: \phi = 1, unit\ root\ is\ present$$

$$H_1: \phi < 1, process\ is\ stationary$$

The test is used on the first difference of the process

$$\Delta y_t = (\phi - 1)y_{t-1} + u_t$$

$$\Delta y_t = \psi y_{t-1} + u_t$$

Which give the hypotheses

$$H_0: \psi = 0, unit\ root\ is\ present$$

$$H_1: \phi < 0, process\ is\ stationary$$

It can also be extended to include an intercept and a deterministic trend

The test statistic is

$$DF = \frac{\hat{\psi}}{se(\hat{\psi})}$$

Which does not follow a standard t-distribution under the null hypothesis because the process is non-stationary under it. Thus, the critical values must be tabulated (using Monte Carlo, for example). Given a significance level α, we reject the null if $DF < C_\alpha$ where C_α is the critical value.

By looking at the Dickey-Fuller results, we see that for each gasoline and gas price level, $DF > C_\alpha$ which implies a failure to reject the null hypothesis. Then, every process should have a unit root, which confirms our intuition. However, what we have not considered is the structural break at around 2008 which can have a significant impact on the conclusion on the DF. So, to do perform this test properly, we must account for this break. This break can be tested using a Chow-test, and then modelled using a TAR. Finally, we must have white noise residuals, and the analysis has not presented any proof of this. I believe that we should instead use the *Augmented Dickey Fuller* model

$$\Delta y_t = \mu + \psi y_{t-1} + \sum_{i=1}^{p} \alpha_i \Delta y_{t-i} + \lambda t + u_t$$

The lags of the process (and perhaps a drift term and a time trend if needed) now control for any autocorrelation in the error term, such that the error term is now white noise. Then the model can be tested using the same test statistic. The choice of lags p can be done by

- Using the frequency of the data as amount of lags
- Using information criteria

Once this is done and the error term can be shown to be white noise, say by using a Q-test, we should repeat the DF test using ADF to actually confirm the order of integration for these models. They are likely to be $I(1)$, but we do not actually know.

Now, we see that the cointegrated analysis is performed using the Engle-Granger method. As previously stated, two $I(1)$ variables are cointegrated if their linear combination is $I(0)$. One linear combination is of course the residuals. So, if we can show that the residuals are $I(0)$, the processes are cointegrated. The analysis does this by estimating the long-run equilibrium regression

$$\ln us_t = \alpha + \beta \ln wti_t + \epsilon_t$$

$$\ln ger_t = a + b \ln brent_t + v_t$$

And collecting their residuals. The residuals are then tested using the outlined Dickey Fuller test. For the U.S we see that

$$DF_{US} = -5.77 < C_{1\%} = -3.43$$

Which implies a rejection of the null and thus the conclusion is that the residuals are $I(0)$ and the U.S gasoline prices and the WTI crude oil are cointegrated. In Germany, we find

$$DF_{US} = -6.6 < C_{1\%} = -3.43$$

Which implies a rejection of the null and thus the conclusion is that the residuals are $I(0)$ and the German gasoline prices and the Brent crude oil are cointegrated. However, we still face the problem of an incorrect specification of the DF model. This DF model of the residuals is simply

$$\epsilon_t = \phi \epsilon_{t-1} + u_t$$

Which should be extended to a model with some variant of the following specification

$$\Delta\epsilon_t = \mu + \psi\epsilon_{t-1} + \sum_{i=1}^{p} \alpha_i \Delta\epsilon_{t-i} + \lambda t + u_t$$

Depending on the behavior of the residuals. One suggestion would be to include the drift and extend p until the error term u_t is white noise. Then, when the break is accounted for, we could perform the rest of the analysis. One "fast" way to perform this analysis is to restrict the data by looking at the period 1996-2007, and then performing the same analysis (but only with an ADF instead of the standard DF model). In this way, we could avoid the problem of modelling the break.

Problem e)

"Using the models learned in class, how would you extend the ECM presented up to now to incorporate the idea of "rockets and feathers"?"

The idea of rockets and feathers is more appropriate in the short-run rather than the long-run, so it makes sense to extend the error correction model to capture this short-run dynamic. This rockets and feathers theory essentially states that positive gasoline price shocks have different effects on future gasoline prices than negative shocks. To quickly sketch this, we can write (regardless of country)

$$\Delta gasoline_t = \alpha_0 + \beta_1 \Delta gasoline_{t-1} + \beta_2 d_1 \Delta gasoline_{t-1} + \epsilon_t$$

Where d_1 is a binary variable that takes the value 1 if $\Delta gasoline_{t-1} > 0$ and 0 else. We can also add a second dummy that takes the value 1 if $\Delta gasoline_{t-1} < 0$ and zero else. Then we have distinguished between the case of positive, negative and no price shock. However, we consider the first (and simpler) case. If we have a positive price shock, then the model becomes

$$\Delta gasoline_t = \alpha + (\beta_1 + \beta_2)\Delta gasoline_{t-1}$$

and if the shock is negative, we will have

$$\Delta gasoline_t = \alpha_0 + \beta_1 \Delta gasoline_{t-1} + \epsilon_t$$

Then, according to this theory β_2 will be large because positive price jumps affect the change in price even more. If needed, we can also extend this idea by including an intercept dummy. Anyway, to incorporate this idea we must go back to the error correction model (regardless of country) and include this dummy. So, one simple error correction model for the U.S could be

$$\Delta \ln us_t = \alpha_0 + \underbrace{\gamma \hat{\epsilon}_{t-1}}_{Long\ run\ equilibrium} + \beta_1 \Delta \ln us_{t-1} + \delta_1 d_1 \Delta \ln us_{t-1} + \beta_2 \Delta \ln wti_{t-1} + \epsilon_t$$

Which can be extended with additional lags (and thus additional dummies). A similar model can be made for Germany.

Problem 1f)

First, the error correction/residual for each country does not seem to be stationary. The error correction for the U.S might be stationary, but with a high variance. However, it seems likely that this is a non-stationary process. In this case, the residuals are not $I(0)$ so we cannot have cointegration. This view is somewhat confirmed by noting that the residuals never converge to zero, which implies that there is no tendency for convergence toward the long run equilibrium. In the U.S, the residuals seem to jump upward for then to be met with a downward jump. One intuitive explanation is that a positive disequilibrium is met with a too strong correction, so the disequilibrium next year will be of the same magnitude, only with a different direction. This process repeats itself and equilibrium is never attained. In Germany, the same affect can be seen to certain degree but with a lower magnitude perhaps. However, it seems like the residuals for Germany are generally on a slight upward trend which indicates a slow divergence *away from equilibrium* where $\hat{\epsilon}_{t-1} = 0$

Problem 1g)

"Kristoufek and Lunackova (2015), and most of this literature, perform the same analysis separately for each country in the dataset. Which shortcoming do you think that this strategy might have?"

One clear shortcoming is that we ignore the simultaneity of the movements in gasoline prices between countries. Gasoline prices might be dependent on prices in other countries and perhaps other exogenous variables (such as exchange rates and interest rates). This is not accounted for when the analysis is performed separately for each country. One solution is to instead consider a VAR based approach where this simultaneous relationship is actually considered. However, for the reasons pointed out in problem c), the analysis would then have to be performed using the Johansen methodology rather than the Engle-Granger method.

Question 2

Problem a)

"The authors write: "We find that public belief in the attainability of the American Dream is not perfectly stable". In the context of a time-series process and using the results presented Figure 4 and in the do/log files, discuss whether you believe this series is stable over time."

The concept of stationarity is closely related to the notion of stability in a time series process. A time series process is *weakly stationary* if the following holds

- $E(y_t) = \mu \ \forall \ t$
- $Var(y_t) = \sigma^2 < \infty \ \forall \ t$
- $Cov(y_{t_1}, y_{t_2}) = \gamma_{t_2 - t_1} \ \forall \ t_1, t_2$

where $\gamma_s = E\{(y_t - E(y_t))(y_s - E(y_{t-s}))\}$ is the autocovariance between y in period t and period $t - k - s$. The last condition states the the autovariance of y at time t and $t + 1$ must equate the covariance of y at time $t + 1$ and $t + 2$. In other words, the covariance structure must be the same for all time periods.

Before we proceed with the answer, we can derive some of the results needed to give us an indication of the process we are looking at in figure 4, and if it is stable. A decaying ACF is a common occurrence for a stationary AR(1) process. We can prove this by finding the general autocorrelation function for an AR(1). It is defined as

$$\rho_k = Corr(y_t, y_{t-k}) = \frac{Cov(y_t, y_{t-k})}{SD(y_t)SD(y_{t-k})}$$

where

$$Cov(y_t, y_{t-1}) = E(y_t y_{t-1}) - E(y_t)E(y_{t-1})$$

The first term can be written as

$$E(y_t y_{t-1}) = E((\mu + \phi_1 y_{t-1} + u_t) y_{t-1})$$

$$E(y_t y_{t-1}) = \mu E(y_{t-1}) + \phi_1 E(y_{t-1}^2)$$

Where the last term is zero because of the white noise assumption. Then, we use the relationship

$$Var(y_t) + E(y_t^2) = E(y_{t-1}^2)$$

So, we get

$$E(y_t y_{t-1}) = \mu E(y_{t-1}) + \phi_1 (Var(y_t) + E(y_t)^2)$$

Insert for the unconditional mean and variance of the AR(1)

$$E(y_t y_{t-1}) = \frac{\mu^2}{1-\phi_1^2} + \frac{\phi_1 \sigma^2}{1-\phi_1^2} + \frac{\phi_1 \mu^2}{(1-\phi_1^2)^2}$$

The stationary assumption guarantees that $E(y_t) = E(y_{t-1})$, so

$$E(y_t)E(y_{t-1}) = \frac{\mu^2}{(1-\phi_1^2)^2}$$

So, the covariance can be written as

$$Cov(y_t, y_{t-1}) = \frac{\mu^2}{1-\phi_1^2} + \frac{\phi_1 \sigma^2}{1-\phi_1^2} + \frac{\phi_1 \mu^2}{(1-\phi_1^2)^2} - \frac{\mu^2}{(1-\phi_1^2)^2}$$

$$Cov(y_t, y_{t-1}) = \frac{\mu^2 + \phi_1 \sigma^2}{1-\phi_1^2} + \frac{\mu^2(\phi_1 - 1)}{(1-\phi_1^2)^2}$$

$$Cov(y_t, y_{t-1}) = \frac{(1-\phi_1)(\mu^2 + \phi_1 \sigma^2) + \mu^2(\phi_1 - 1)}{(1-\phi_1^2)^2}$$

Change the sign on the last term

$$Cov(y_t, y_{t-1}) = \frac{(1-\phi_1)(\mu^2 + \phi_1 \sigma^2) - \mu^2(1-\phi_1)}{(1-\phi_1^2)^2}$$

$$Cov(y_t, y_{t-1}) = \frac{\phi_1 \sigma^2}{1-\phi_1^2}$$

Then the autocorrelation can be written as

$$Corr(y_t, y_{t-1}) = \frac{\frac{\phi_1 \sigma^2}{1-\phi_1^2}}{\frac{\sigma^2}{1-\phi_1^2}} = \phi_1^1$$

This can be generalized such that

$$Corr(y_t, y_{t-p}) = \phi_1^p$$

Then, if the process is stationary and coefficient $0 < \phi_1 < 1$ and the ACF will strictly decay from above. This is what we see in the ACF plot. So, this process will at the very least have some AR behavior that is stable. This means that the belief in the American dream is partially based its previous realizations. Such a result does make sense as you would expect optimism among Americans to spread and also reinforce their beliefs (similarly when they are pessimistic) Generally, when presented with an ACF and the PACF, the order of the AR process (assuming it is stationary) is determined by the order of significant partial autocorrelation coefficients. The partial autocorrelation measures the correlation between the observation of $amdream_t$ with the observation of $amdream$, k periods backward, that is, $amdream_{t-k}$ after controlling for all lags before the k'th lag. If we estimate an $AR(3)$ such as

$$amdream_t = \phi_0 + \phi_1 amdream_{t-1} + \phi_2 amdream_{t-2} + \phi_3 amdream_{t-3} + u_t$$

Then the parameters ϕ_1, ϕ_2, ϕ_3 gives the partial autocorrelation between $amdream_t$ and $amdream_{t-1}, amdream_{t-2}, amdream_{t-3}$ respectively. Lags beyond 3 will in this case not be considered and the partial autocorrelation will be 0. The figure shows that the first lag has a high autocorrelation of over 0.8. Beyond this, there are four significant autocorrelation terms where two of them are marginally significant. Then, there are two more significant coefficients which I assume can come from some political events. This suggests that we are dealing with an AR(1) component where the two events should be controlled for using a dummy variable. However, we should take note that an $ARMA(p\ , q)$ process will have

- Geometrically decaying autocorrelation and partial autocorrelation
- The autocorrelation will decay (either directly or oscillatory) after lag q.
- The partial autocorrelation will decay (either directly or oscillatory) after lag p.

By looking at the ACF and PACF, we could also argue for an ARMA(1,1), alternatively some higher order MA process. However, the parsimony of both the AR and ARMA process relative to a higher MA order process suggests that this MA process can be ignored. Anyway, the ACF and PACF alone suggests that we are looking at a stationary ARMA(1,1) or AR(1). However, we might want to test whether these models have white noise residuals. So, the belief in the American Dream should be stable over time if we believe the ACF and PACF.

Problem b)

"They estimate the model on line 32 of the do-file and 22 of the log-file. Discuss the results"

The estimated model is

$$\Delta amdream_t = -0.25 - 343.8\Delta ginihq_{t-1} + 0.977\Delta homeown_{t-1} - 0.186\Delta mood_{t-1} + 0.486prezcamp + 1.372midterm$$

What we can note immediately is that none of these estimates are statistically significant at $\alpha = 5\%$, so this is likely to be a bad model specification. In addition, the results are hard to interpret. From the text it is not clear on which unit the variable $amdream$ is measured as which complicates the interpretation of the coefficients. Looking away from the issue of significance, the model estimates are somewhat reasonable in terms of the signs of parameters. A positive increase in the Gini coefficient (higher inequality) causes American to lower their beliefs in the American dream, as seen by a negative change in $\Delta amdream_t$. We also note that this coefficient is large in absolute value, so apparently inequality has significant impacts on the belief of the American dream relative to the other variables. You could perhaps perform some mental gymnastics and argue that higher inequality motivates people to work hard, so the coefficient should be positive. Anyway, this is likely not the case. If the homeownership increases by one percentage point, the belief in the American dream increases. This makes sense. Increased optimism (assuming mood is related to consumer sentiment) of the economy in the short term has a negative impact on the belief of the American dream. This does not necessarily make sense.

Finally, the measures of campaign rhetoric seem to indicate that Americans tend to believe more in the American dream on average (captured by the intercept) when we are in either midterms or in a presidential election year. This makes sense as people tend to believe all the lies told by politicians about future prosperity and improvement of welfare in the country. To conclude, the lack of any strong results are perhaps due to a bad specification. Measurement error in terms of $amdream_t$ can also contribute to poor results as measurement error widens the confidence interval of the coefficient estimates.

Problem c)

Their estimation strategy suffers from poor model selction. The answers from a) indicate that we have some sort of AR or ARMA process which is entirely ignored in b).

The answer to this question depends on what the purpose of the analysis is. If we want to identify the determinants of the American dream, one suggestion is the following:

First, we can test if $amdream$ variable is stationary using an ADF. I will not specify the procedure as I already did this in question 1. If it is $I(1)$ we must look at $\Delta amdream_t$ instead. If not, we can simply proceed with using $amdream$.

When this is done, we estimate both an ARMA(1,1) and an AR(1) and test their residuals for autocorrelation using a Q-test. This is conducted by formulating the hypotheses

$$H_0: \rho_1 = \cdots = \rho_m = 0 \; against \; H_1: not \; H_0$$

The test statistic is

$$Q - stat = T(T+2)\sum_{k=1}^{m} \frac{\hat{\rho}_k^2}{T-k} \sim \chi_m^2$$

The null is rejected when $Q - stat > \chi_m^2$. Here, we could test different values of m. We should probably look at $m > 10$ or 20.

If we are interested in using the variables presented in b), we add them to both the AR and ARMA model and see if they yield any results. However, the variables we considered in b) must be tested for stationarity using an ADF if we are to use them here. The simple DF test in the analysis suggests that the Gini and homeownership variables are stationary. The consumer sentiment mood is not tested however, so we should do this.

When this is done and we use the stationary variables, we add them to the models and test for zero autocorrelation again using the Q-test. If the null for one model is rejected while the other model has a null that cannot be rejected, we simply pick the model that has the white noise errors. If not, we can use the information criteria AIC and BIC to give us a suggestion for which model is the most effective in capturing the dynamics of the data. The information criteria are

$$AIC = \ln \hat{\sigma}^2 + \frac{2k}{T}$$

$$BIC = \ln \hat{\sigma}^2 + \frac{k}{T} \ln T$$

Where k is the number of estimated parameters and $\hat{\sigma}^2$ is the SSR. In this case, a lower score from the AIC and BIC is better because the parameters of the model are more effective in jointly reducing the SSR than the model with the higher AIC/BIC. Finally, when the AR and ARMA is estimated, we should check if their stability conditions hold. For a general ARMA(p, q)

$$y_t = \mu + \sum_{i=1}^{p} \phi_i y_{t-i} + \sum_{i=1}^{q} \theta_i u_{t-i} + u_t$$

Stability is attained when

$$\sum_{i=1}^{p} \phi_i < 1$$

So, for an AR(1) and ARMA(1,1) we would like

$$\phi_1 < 1$$

If this holds, the model is stable.

If we instead wanted to forecast $amdream_t$, we should probably drop the other variables (Gini, homeownership, mood, campaign dummies) because forecasts tend to perform better when the number of parameters is low.