Problem 1

a) Face value = 100, time to maturity = 3, $r_f = 0,03$

$P_0^G = P_T \cdot e^{-r_f t} = 100 \cdot e^{-0,03 \cdot 3} = \underline{\underline{91,39}}$

b) $\lambda = 0,05$

Default probability (unconditional): $Q(t) = 1 - e^{-\lambda \cdot t}$

$Q(t) = 1 - e^{-0,05 \cdot 3} = 0,1393$, $Q(t) = PD$

The default probability is 13,93%

c) $R = 0,6 \Leftrightarrow LGD = 1 - R = 0,4$

Time 0 value of the bond:

$$P_0^C = (1 - PD) \cdot P_T \cdot e^{-rT} + (1 - LGD) \cdot PD \cdot P_T e^{-rT}$$

$$= (1 - 0,1393) \cdot 100 \cdot e^{-0,03 \cdot 3} + (1 - 0,4) \cdot 0,1393 \cdot 100 e^{-0,03 \cdot 3}$$

$$= 78,6621 + 7,6386$$

$P_0^C = \underline{\underline{86,3}}$

d) ~~yield~~ $= \cancel{\frac{\ln\left(\frac{1}{P_0/100}\right)}{T}} = \cancel{\frac{\ln\left(\frac{1}{86,3}\right)}{3}} =$

$$\text{yield} = \frac{\ln\left(\frac{1}{P_0/100}\right)}{T} = \frac{\ln\left(\frac{1}{P_0/P_T}\right)}{T} = \frac{\ln\left(\frac{1}{86,3/100}\right)}{3}$$

$= \underline{\underline{0,049}}$, Spread $= y - r = 0,049 - 0,03 = 0,019$.

Can also calculate the credit spread (S):

$S = \frac{LGD \cdot PD}{T} = \frac{0,4 \cdot 0,1393}{3} = 0,0186$

yield $= r_f + S = 0,03 + 0,0186 = \underline{\underline{0,049}} = \underline{\underline{4,9\%}}$

spread $= 0,0186 = \underline{\underline{1,86\%}}$ which is about the same as yield $- r_f = 0,049 - 0,03 = \underline{0,019}$

Task 2)

a) Working capital (WC) = inventory + Trade debtors − Trade Creditors

$$WC\ 2017 = 45 + 17 - 32 = \cancel{94}\ \underline{30}$$

$$WC\ 2018 = 43 + 22 - 34 = \cancel{99}\ \underline{31}$$

$$WC\ 2019 = 42 + 14 - 28 = \cancel{84}\ \underline{28}$$

The WC increased from 2017 to 2018, and further decreased from 2018 to 2019.

b) Inventory holding period (IHP) = ~~Inventory current year~~

$$= \frac{\text{inventory}}{\text{cost of sales}} \times 365$$

I assume end of year value of inventory equals average value.

$$IHP\ 2017 = \underline{80} = (45/204) \times 365$$

$$IHP\ 2018 = \underline{74} = (43/212) \times 365$$

$$IHP\ 2019 = \underline{74} = (42/207) \times 365$$

After 2017, the firm has managed to reduce the holding period by six days. This is good

c) Debtors collection period = (DCP) = $\frac{\text{Trade debtors}}{\text{Sales}} \times 365$

$$DCP\ 2017 = (17/265) \times 365 = \underline{23{,}42}$$

$$DCP\ 2018 = (22/284) \times 365 = \underline{28{,}27}$$

$$DCP\ 2019 = (14/275) \times 365 = \underline{18{,}58}$$

I have used the total sales since there is no information about the credit sales in particular. I therefore assume that all sales are credit sales, which might be realistic since doors and windows are often order items.

Task 2 c) continued

The firm has managed to reduce the debtors collection period the last year, which is good, because it indicates efficiency from for example good costumers.

d) Creditors payment period = (CPP) = $\frac{\text{Trade creditors}}{\text{cost of sales}} \times 365$

CPP 2017 = $\frac{\cancel{17}\,32}{204} \times 365 = \underline{57,25}$

CPP 2018 = $\frac{\cancel{22}\,34}{212} \times 365 = \underline{58,54}$

CPP 2019 = $\frac{28}{207} \times 365 = \underline{49,37}$

The firm has reduced CPP the last year. This means that they on average have shorter time to pay the creditors, which is not desireable. It could however also indicate that the firm buys less on credit, which might be because of better liquidity, which is good.

e) Cash cycle = operating cycle − creditors payment period

Operating cycle = inventory holding period + debtors collection period

Cash cycle 2017 = $(80 + \cancel{57,25}\ 23,42) - 57,25 = \underline{46,17} \approx 46$

Cash cycle 2018 = $(74 + 28,27) - 58,54 = \underline{43,73} \approx 44$ days

Cash cyle 2019 = $(74 + 18,58) - 49,37 = \underline{43,21} \approx 43$ days

The firm has reduced the cash-cycle period from 46 days in 2017 to 43 days in 2019. So they have managed to lower this, but from 2018 to 2019 it barely changed, so they could probably lower it more by for example reducing the inventory holding period or increasing the creditors payment period

Task 3)

- For a bank that lends money to different debitors, there exists maturity risk. This rises because the time to maturity differs, and the risk is that at a point in time, ~~they~~ the bank might not have enough liquidity to pay what they owe to their creditors. Even though the bank has lent out an amount of money that is to be paid back in the future, they also need cash in ~~case of~~ ~~sudden~~ order to pay their invoices

- ~~To manage~~ To manage this risk, the bank should spread out the maturity dates so that they have smaller amounts coming in regularly, rather than big amounts that are ~~paid~~ recieved on the same day. Then, they will make sure to always have enough liquidity to pay there commitments at due date.

Another way to manage maturity risk for the bank, is to always make sure it is possible to borrow money in the market for a short amount of time, until they recieve enough to have a stable liquidity in the future. In the case of a financial crisis, the opportunity to borrow in the market is reduced, and so the maturity risks for the banks increases.

Problem 3b)

Wrong way counterparty risk exists when exposure to a counterparty is negatively correlated with the credit quality of this counterparty.

~~B~~ For example as a buyer of a put option on a stock, you are exposed to the company that this stock represents, and you will earn money if the credit quality of that company lowers (because then the stock price would most probably lower as well, and you will exercise the option).

If the credit exposure of the put option buyer to their counterparty (here: the put option seller) increases ~~as~~ at the lifetime of the put option, then the buyer is exposed to wrong-way risk.

This risk could arise from macroeconomic factors, as we saw in the financial crisis of 2008, when the creditworthines of banks ~~fell~~ was reduced.

It could also arise from specific factors within the counterparty, for example reduced credit grade due to lower earnings.

this kind of risk is one of the problems that the Basel III ~~B~~ will try to identify.

Problem 4,

- The Leland model: $V_0 = 100$, $\sigma = 0{,}2$, $r_f = 0{,}05$

$\tau = 0{,}25$, perpetual debt: $T - t = \infty$, $C = 5$, $\alpha = 0{,}5$

a) Optimal default barrier: $V_B = (1-\tau) \cdot \dfrac{C}{r + \frac{1}{2}\sigma^2}$

$$V_B = (1-0{,}25) \cdot \frac{5}{0{,}05 + \frac{1}{2} \cdot (0{,}2)^2} = \underline{\underline{53{,}57}}$$

b) Debt value $= F(V)$. The Leland model:

- $F(V) = C \cdot \pi_2(V) + (1-\alpha) \cdot V_B \cdot \pi_1(V)$

where $\pi_1(V) = \left(\dfrac{V}{V_B}\right)^{-X}$ and $\pi_2(V) = \dfrac{1}{r} \cdot (1 - \pi_1(V))$

and $X = \dfrac{2r}{\sigma^2} = \dfrac{2 \cdot 0{,}05}{0{,}2^2} = 2{,}5$

$$\pi_1(V) = \left(\frac{100}{53{,}57}\right)^{-2{,}5} = \frac{\overset{0{,}2100}{\cancel{3{,}4846}}}{}, \quad \pi_2(V) = \frac{1}{0{,}05} \cdot (1 - \underset{0{,}2100}{\cancel{3{,}4846}}) = \cancel{49{,}6926} = 15{,}8$$

Insert into Leland's model:

$\cancel{F(V) = 5 \cdot (49{,}6926) + (1-0{,}5) \cdot 53{,}57 \cdot 3{,}4846} =$

- $F(V) = 5 \cdot 15{,}8 + (1-0{,}5) \cdot 53{,}57 \cdot 0{,}21 = \underline{\underline{84{,}6249}}$

Problem 4 c)

Value of equity : $E(V) = v(V) - F(V)$

$$E(V) = V + \tau \cdot C \cdot \pi_2 - \alpha \cdot V_B \cdot \pi_1 - F(V)$$

$$= 100 + 0{,}25 \cdot 5 \cdot 15{,}8 - 0{,}5 \cdot 53{,}57 \cdot 0{,}21 - 84{,}6249$$

$$= \underline{\underline{29{,}5}}$$

d) Optimal Coupon $C^* = V\left((1+x) \cdot h\right)^{-\frac{1}{x}}$

$$h = \left(1 + x + \alpha(1-\tau)x\right) \cdot m$$

$$= 1 + 2{,}5 + 0{,}5(1-0{,}25) \cdot 2{,}5) \cdot m = 4{,}4375 \cdot m$$

$$m = \frac{(1-\tau)x}{r(1+x)^x/(1+x)} = \frac{(1-0{,}25) \cdot 2{,}5}{0{,}05(1+2{,}5)^{2{,}5}/(1+2{,}5)} = 5{,}7270$$

$$\Rightarrow h = 4{,}4375 \cdot 5{,}727 = 25{,}4137$$

$$\Rightarrow C^* = 100\left((1+2{,}5) \cdot 25{,}4137\right)^{-\frac{1}{2{,}5}} = \underline{\underline{16{,}61}}$$

e) Insurance that pays 100 at the time of default

The price should be $100 \times \left(\frac{V}{V_B}\right)^{-x} = 100 \times \pi_1(V)$

$$= 100 \times 0{,}21 = \underline{\underline{21{,}00}}$$

f) ~~In general, the credit risk increases as T increases, and so the credit spread should also increase.~~

Problem 4 f)

Merton defines credit spread (H) as

$$H = \frac{1}{(T-t)} \cdot \ln\left(N(d_2) + \frac{V}{D \cdot B(t,T)} \cdot N(-d_1)\right)$$

~~as a bank that issues a perpetual loan,~~

if using Merton's model to calculate the credit spread, we see from this formula that if $T \to \infty$, then $(T-t) \to \infty$, and the first term, $\frac{1}{(T-t)}$ approaches zero. This cancels out the whole formula, and so for a perpetual loan, the credit spread is zero.

Problem 4g)

$$PD = P(V \leq V_B)$$

On a computer, we could use Monte Carlo simulations to estimate the probability of default.
The value that changes in each simulation is the time to maturity, T
Here, T_x indicates the T on a given simulation, x.
We create a value $A_1 : V_{T_x} \leq V_B$ = default
and $\bar{A}_1 : V_{T_x} > V_B$ = survival/solvent.
For each simulation, if default, A_1 will have value 1, and if solvent, $\bar{A}_1$ will have value 1 (and $A_1 = 0$)

Create a loop: For $i=1$, $i \leq N$, i++

$$\{ Sum = Sum + 1 \cdot A_1 \cdot e^{-r \cdot T_x} + 1 \cdot \bar{A}_1 \cdot e^{-r \cdot T_x}$$

$$\{ PD = \frac{Sum}{N}$$

Sum represents the sum of all previous simulations.
In each loop, we find the discounted PD, and then we find the average of all of these sums.
This average represents the probability of default.

The more simulations, the more accurate will the PD be, but this also takes time. A number of 1000 or 10000 simulations would probably be good in this case.