Exam SØK1001 høst 2016

Oppg. 1 (30%)

a) i) $f(x) = \dfrac{x^3+5x}{2x^2-1}$

$$f'(x) = \frac{(3x^2+5)(2x^2-1) - (x^3+5x)4x}{(2x^2-1)^2}$$

$$= \frac{6x^4 - 3x^2 + 10x^2 - 5 - 4x^4 - 20x^2}{(2x^2-1)^2}$$

$$= \frac{2x^4 - 13x^2 - 5}{(2x^2-1)^2}$$

ii) $f(x) = \left(\frac{1}{4}x^3 - \ln x\right)^4$

$$f'(x) = 4\left(\frac{1}{4}x^3 - \ln x\right)^3 \cdot \left(\frac{3}{4}x^2 - \frac{1}{x}\right)$$

$$= \left(\frac{1}{4}x^3 - \ln x\right)^3 \left(3x^2 - \frac{4}{x}\right)$$

iii) $f(x) = x^2(e^{4x} - 2)$

$$f'(x) = 2x(e^{4x} - 2) + x^2 \cdot e^{4x} \cdot 4$$

$$= 2x(e^{4x} - 2) + 4x^2 e^{4x}$$

b) i) $P(t) = 6 \cdot 1{,}0075^t$

ii) $6 \cdot 1{,}0075^t = 12$

$1{,}0075^t = 2$

$\ln 1{,}0075^t = \ln 2$

$t = \dfrac{\ln 2}{\ln 1{,}0075} \approx \underline{92{,}8 \text{ år}}$

iii) $6 \cdot 1{,}0075^t = 9$

$t = \dfrac{\ln\left(\frac{3}{2}\right)}{\ln 1{,}0075} \approx \underline{54{,}3 \text{ år}}$

c) i) $f(x) = \frac{2}{3}x^3 + 3x^2 - 1$

$f'(x) = 2x^2 + 6x$

$f'(0) = 0$

$f(0) = -1$

$y - f(0) = f'(0)(x-0)$

$y - (-1) = 0(x-0) \Rightarrow \underline{y = -1}$

ii) $g(x) = \ln x^3 + 2x^4$

$$g'(x) = \frac{1}{x^3} \cdot 3x^2 + 8x^3$$

$$= \frac{3}{x} + 8x^3$$

$g'(1) = 11$

$g(1) = 2$

$y - g(1) = g'(1)(x-1)$

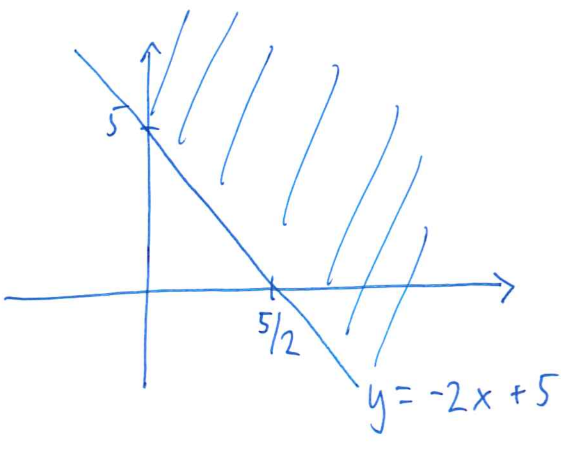
$y - 2 = 11(x-1)$

$y = 11x - 11 + 2$

$\underline{y = 11x - 9}$

d) $f(x,y) = \dfrac{1}{\sqrt{2x+y-5}}$

D_f: $2x + y - 5 > 0 \Rightarrow y > -2x + 5$



$D_f = ///$

(linja $y = -2x+5$ er ikke del av D_f)

Oppg. 2 (20%)

$$f(x) = -\frac{1}{3}x^3 - \frac{1}{2}x^2 + 2x + \frac{4}{3}$$

a) $f'(x) = -x^2 - x + 2$

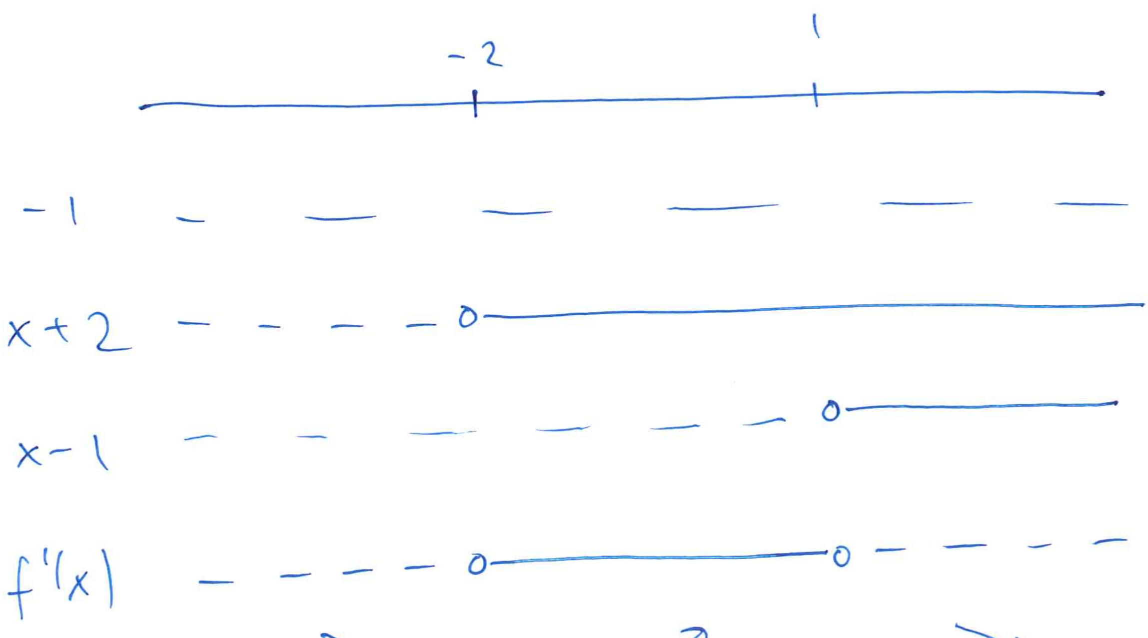
$f''(x) = -2x - 1$

b) $f'(x) = 0$

$$\Rightarrow x = \frac{1 \pm \sqrt{1 - 4 \cdot (-1) \cdot 2}}{2 \cdot (-1)}$$

$$= \frac{1 \pm 3}{-2} \qquad x_1 = -2 \quad \text{og} \quad x_2 = 1$$

$$f'(x) = -(x+2)(x-1)$$



$$f(-2) = -\frac{1}{3} \cdot (-2)^3 - \frac{1}{2}(-2)^2 + 2 \cdot (-2) + \frac{4}{3}$$

$$= \frac{8}{3} - 2 - 4 + \frac{4}{3}$$

$$= -6 + 4 = -2$$

$$f(1) = -\frac{1}{3} - \frac{1}{2} + 2 + \frac{4}{3}$$

$$= \frac{5}{2}$$

Lokalt min: $(-2, -2)$

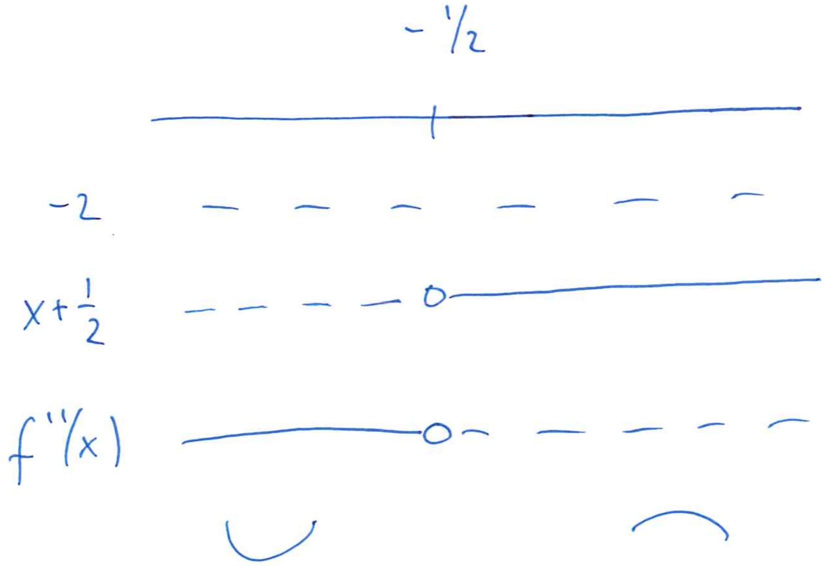
Lokalt max: $(1, 5/2)$

c) $f''(x) = 0$

$$-2x - 1 = 0$$

$$2x = -1$$

$$\underline{x = -\frac{1}{2}}$$



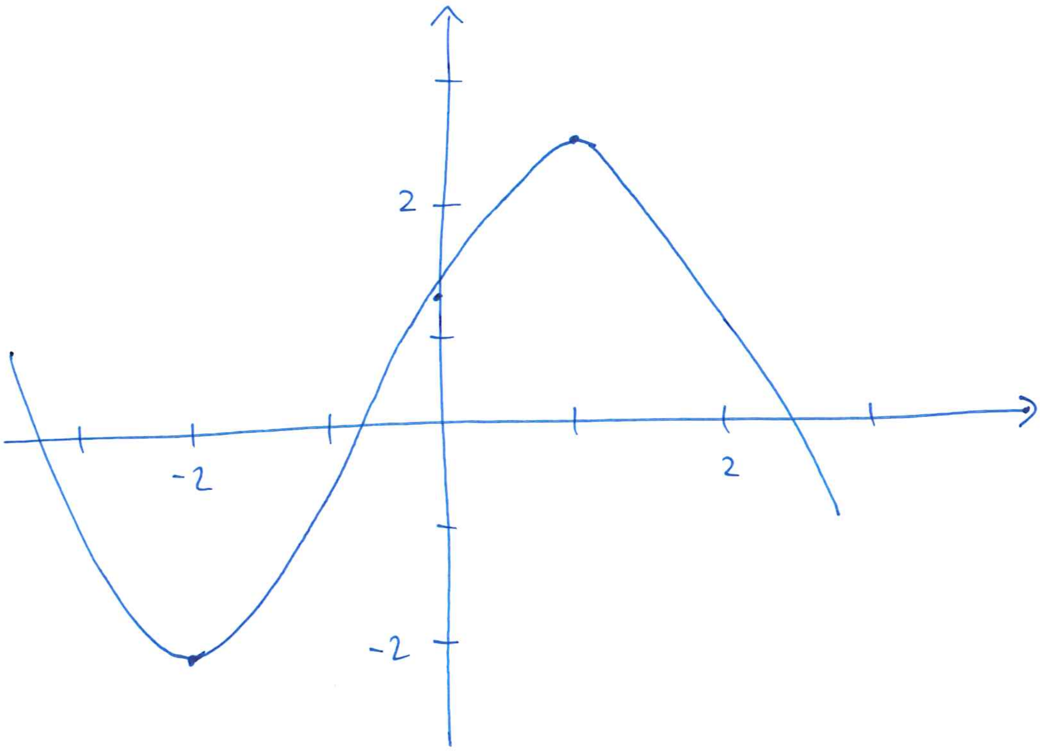
$$f''(x) = -2x - 1$$

$$= -2\left(x + \frac{1}{2}\right)$$

Vendepunkt for $\underline{\left(-\frac{1}{2}, 1/4\right)}$

$$f\left(-\frac{1}{2}\right) = \left(-\frac{1}{3}\right) \cdot \left(-\frac{1}{8}\right) - \frac{1}{2} \cdot \frac{1}{4} + 2 \cdot \left(-\frac{1}{2}\right) + \frac{4}{3} = \frac{1}{24} - \frac{1}{8} - 1 + \frac{4}{3} = \frac{1}{24} - \frac{3}{24} + \frac{8}{24} = \frac{6}{24} = 1/4$$

d)



Opps. 3 (15%)

$$f(x,y) = \frac{1}{3}x^3 + 6y^2 - 6xy - 10x$$

$$f_1'(x,y) = x^2 - 6y - 10$$

$$f_2'(x,y) = 12y - 6x$$

$$x^2 - 6y - 10 = 0 \qquad \text{(i)}$$

$$12y - 6x = 0 \qquad \text{(ii)}$$

(ii) gir $12y = 6x \Rightarrow y = \frac{1}{2}x$ inn i (i):

(i) $x^2 - 6 \cdot \frac{1}{2}x - 10 = 0$

$$x^2 - 3x - 10 = 0$$

$$x = \frac{3 \pm \sqrt{9 - 4 \cdot 1 \cdot (-10)}}{2 \cdot 1}$$

$$= \frac{3 \pm 7}{2}$$

$x_1 = 5$ og $x_2 = -2$

$\Downarrow$ $\qquad$ $\Downarrow$

$y = \frac{5}{2}$ $\qquad$ $y = -1$

To stasjonære punkt $(x,y) = (5, 5/2)$ og $(x,y) = (-2,-1)$.

Andrederiverttest:

$f''_{11}(x,y) = 2x \quad = A$

$f''_{12}(x,y) = -6 \quad = B$

$f''_{22}(x,y) = 12 \quad = C$

(x,y)	A	B	C	$AC-B^2$	Type punkt
$(5, 5/2)$	10	-6	12	84	Lokalt min
$(-2,-1)$	-4	-6	12	-84	Sadelpunkt

Oppg. 4 (15%)

$$\frac{1}{2}x^2 - xy + 2y^2 = 6$$

a) Skjæring 1. aksen: $y=0$

$$\Rightarrow \frac{1}{2}x^2 = 6 \Rightarrow x^2 = 12 \Rightarrow x = \pm\sqrt{12}$$

Skjæring 2. aksen: $x=0$

$$\Rightarrow 2y^2 = 6 \Rightarrow y^2 = 3 \Rightarrow y = \pm\sqrt{3}$$

b) $x - (y + xy') + 4yy' = 0$

$$x - y - xy' + 4yy' = 0$$

$$(4y-x)y' = y - x$$

$$y' = \frac{y-x}{4y-x}$$

c) $y' = 0$ gir $y = x$

I tillegg må punktet ligge på kurva:

$$\frac{1}{2}x^2 - xy + 2y^2 = 6$$

$$\Rightarrow \frac{1}{2}x^2 - x^2 + 2x^2 = 6$$

$$\Rightarrow \frac{3}{2}x^2 = 6$$

$$\Rightarrow x^2 = 4$$

$$\Rightarrow x = \pm 2$$

Tangenten er horisontal i punkta $(2,2)$ og $(-2,-2)$.

y'':

Tar utgangspunkt i $x - y - xy' + 4yy' = 0$

$$1 - y' - (y' + xy'') + 4y'y' + 4yy'' = 0$$

$$1 - y' - y' - xy'' + 4y'y' + 4yy'' = 0$$

$$\Rightarrow 1 - 2y' + 4y'y' + (4y-x)y'' = 0$$

$$y'' = \frac{2y' - 4y'y' - 1}{4y-x}$$

I punktet $(2,2)$ med $y'=0$:

$$y'' = \frac{-1}{6} < 0 \qquad \text{Konkav}$$

I punktet $(-2,-2)$ med $y'=0$:

$$y'' = \frac{-1}{-6} = \frac{1}{6} > 0 \qquad \text{Konveks}$$

Oppg. 5 (20%)

a) $\mathcal{L} = 2x + y - \lambda(y - x^2 + 4x - 2)$

FOB:

$\mathcal{L}_1' = 0 \Rightarrow 2 - \lambda(-2x+4) = 0$ (i)

$\mathcal{L}_2' = 0 \Rightarrow 1 - \lambda = 0$ (ii)

$y - x^2 + 4x = 2$ (iii)

(ii) gir $\lambda = 1$ inn i (i):

$$2 - (-2x + 4) = 0$$

$$2 + 2x - 4 = 0$$

$$2x = 2$$

$$\underline{x = 1}$$

Fra (iii): $y = x^2 - 4x + 2$

$$= 1 - 4 + 2$$

$$\underline{= -1}$$

Optimalt punkt: $(x, y) = (1, -1)$ med funksjonsverdi:

$$f(1, -1) = 2 \cdot 1 + (-1)$$

$$= 1$$

b) Nivåkurver:

$f(x,y) = -1 \Rightarrow 2x + y = -1 \Rightarrow y = -2x - 1$

$f(x,y) = 1 \Rightarrow 2x + y = 1 \Rightarrow y = -2x + 1$

$f(x,y) = 3 \Rightarrow 2x + y = 3 \Rightarrow y = -2x + 3$

→ Rette linjer med stigningstall lik -2.

Bibetingelsen:

$y - x^2 + 4x = 2 \Rightarrow y = x^2 - 4x + 2$

→ Konveks 2. gradsfunksjon

Bunnpunkt: $y' = 0 \Rightarrow 2x - 4 = 0 \Rightarrow x = 2$

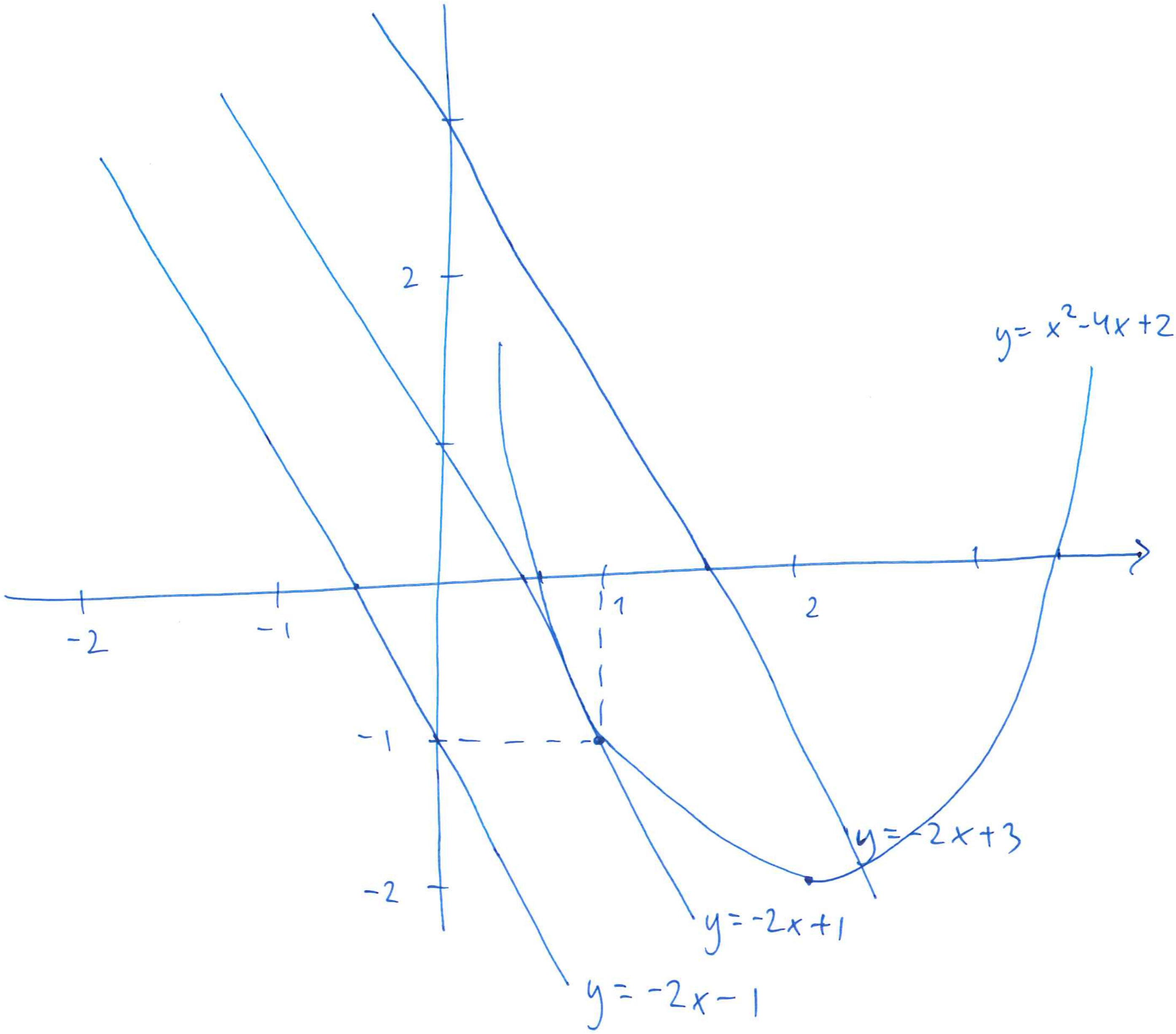
→ I punktet $(2, -2)$

Skjæring 1. aksen:

$y = 0 \Rightarrow x^2 - 4x + 2 = 0$

$$x = \frac{4 \pm \sqrt{16 - 4 \cdot 2}}{2} = \frac{4 \pm \sqrt{8}}{2}$$

$$x_1 = \frac{4 + \sqrt{8}}{2} \approx 3,4 \qquad x_2 = \frac{4 - \sqrt{8}}{2} \approx 0,6$$



Punktet $(1, -1)$ er et min punkt. Det er ikke mulig å komme på en lavere nivåkurve enn $f(x, y) = 1$.