

Question 1

They should never change their choice to Y since it breaks the rule about transitivity. Transitivity states that if the individual in this example prefer X over Y and an introduction of Z, an inferior choice comes in should not change the individuals' choice. Let's say that X is better than Y and Y is better than Z. The individual is stated to be rational, so he must then prefer X over Z. This assumes that the individual has preferences over X, Y and Z. Then the individual has fulfilled the rule about completeness. But in the real world, people will maybe choose Y, even if it is irrational. This is called the decoy effect or the attraction effect. In this example, the introduction of Z makes the individual chose Y even though the individual has a strictly preference X over Y. This violated the expansion condition which is build up from the assumption about transitivity. Even if choice Z is better than X, the individual should not change to Y.

Question 2

		Taxpayer	
		Pay Tax	Cheat
Tax Authority	Audit	3,0	5,-12
	Not Audit	5,0	0,5

Nash equilibrium is a strategy where each of the individuals' choice is the best response given the other individual's choice. Best response is that the individual choice is the best he/she can choose given what has been chosen by the other person.

Let's look at the tax payers best response to the tax authority:

Tax authority chooses Audit, then taxpayer chooses Paying taxes, since paying taxes gives utilities 0 rather than -12 from cheating. This gives (Audit, Pay tax).

Tax authority chooses Not Audit, then taxpayer chooses Cheating, since cheating gives utility 5 rather than 0 from paying taxes. This gives (No Audit, Cheat).

Let's look at the tax authority best response to the taxpayer:

Taxpayer chooses Pay tax, then tax authority chooses No audit, since no audit gives utility 5 rather than 3 from audit. This gives (No Audit, Pay tax).

Taxpayer chooses Cheat, then tax authority chooses audit, since paying audit gives utility 5 rather than 0 from no audit. This gives (Audit, Cheat).

From the responses, we see that none are the same. This means that there are no Nash equilibria in pure strategies. One individual's best response to the other individual's choice isn't the same response the other way around. The individual is better off choosing something different.

Let's look at mixed strategies.

Tax authority has a chance q of getting 3 and a chance of $1-q$ of getting 5. We can therefore find the utility for each choice given the chances.

$$U(\text{Audit}) = q * 3 + (1 - q) * 5$$

$$U(\text{No audit}) = q * 5 + 0 * (1 - q)$$

Assume that tax authority is indifferent between its choices.

$$U(\text{Audit}) = U(\text{No audit})$$

Solve for q :

$$q * 3 + (1 - q) * 5 = q * 5 + 0 * (1 - q)$$

$$-2q - 5q = -5$$

$$q = \frac{5}{7}$$

Doing the same for taxpayer:

$$U(\text{Pay tax}) = p * 0 + (1 - p) * 0$$

$$U(\text{Cheat}) = p * -12 + 5 * (1 - p)$$

$$p * 0 + (1 - p) * 0 = p * -12 + 5 * (1 - p)$$

$$12p + 5p = 5$$

$$p = \frac{5}{17}$$

There is a Nash equilibrium in mixed strategies. The tax authority will do an audit with a probability of 5/17 and the taxpayer will pay taxes with a probability of 5/7.

Question 3

a)

Using the value function from the book:

$$v(x) = \sqrt{\frac{x}{2}} \text{ for gains } (x \geq 0)$$

$$v(x) = -2\sqrt{|x|} \text{ for losses } (x < 0)$$

Assume that I segregate the losses and gains, since in task b I will look at integrating the two choices.

Lets look at a situation where I look from the initial position before I win. My initial position will be 0. The gains of 50000 will be $v(+50000)$, while the tax will be $v(-20000)$, $50000 \cdot 0,4$ is 20000. Plot in the numbers.

$$v(+50000) = \sqrt{\frac{50000}{2}} = \sqrt{25000} = 158,11$$

$$v(-20000) = -2\sqrt{20000} = -282,84$$

My value if I segregate and looking from initial position will be

$$\begin{aligned} &v(+50000) + v(-20000) \\ &= 158,11 - 282,84 \\ &= -124,73 \end{aligned}$$

If an individual start from initial position of 0 and segregate the gains and losses, the individual will feel that he/she has lost. The individual value losses more than gains, and therefore the tax on the winnings has a bigger impact than the winning of 50000.

What is the person looked from the best position, starting from 50000 and then has to pay taxes. Initial position is 50000. Using the value function the tax is a loss of 20000 from the initial position.

$$v(-20000) = -2\sqrt{20000} = -282.84$$

The value of looking from the best position is larger than looking from initial position of 0 in terms of negative effect. Here we see that individuals have different views of losses and gains. $-124,73$ compared to $-282,84$. Starting from 50000 will feel like a larger loss than if starting from 0. The individual is less dissatisfied when looking from position 0.

b)

Let's see what happens if the individual integrates the gains and losses. Assume starting from 0. Integrating means the individual says he/she has won 30000 rather than winning 50000 and then "losing" 20000.

$$v(+50000 - 20000) = v(+30000)$$

$$v(+30000) = \sqrt{\frac{30000}{2}} = \sqrt{15000} = 122,47$$

Here we see that the individual will get a value of 122,47 if he/she were to mentally bundle winning and paying tax into one amount. Compared to segregating it is better to integrate gains and large losses into one amount. The individual won't feel the large loss by integrating the gain and the tax into one.

Question 4

	t=0	t=1	t=2
A	0	2	8
B	0	5	0

a) Exponential discounter is a model that wants to capture what money in the future is worth to you today. Martina being an exponential discounter means that she discounts money in the future by $\delta = 0.6$ and is time consistent. Her discounting doesn't change over

time. A krone tomorrow is worth $u * \delta$ for Martina. Using the delta function for Martina which is given as:

$$U^0(u) = u_0 + \sum_{i=1}^2 \delta^i * u_i$$

I will use the delta function for Martina looking from $t=0$. Assume that in $t=0$ she gets 0 utilities. Plotting in the number to find utility of choice a and b:

$$\begin{aligned} U^0(u) &= 0 + \delta^1 u_1 + \delta^2 u_2 \\ U^0(a) &= 0 + 0.6 * 2 + 0.6^2 * 8 \\ U^0(a) &= 4.08 \\ U^0(b) &= 0 + 0.6 * 5 + 0.6^2 * 0 \\ U^0(b) &= 3 \\ U^0(a) &> U^0(b) \end{aligned}$$

In time $t=0$ she will want to choose A since it gives a higher utility.

Time $t=1$. Using the same function but change it since $t=1$.

$$\begin{aligned} U^1(u) &= u_1 + \delta u_2 \\ U^1(a) &= 2 + 0.6 * 8 \\ U^1(a) &= 6,8 \\ U^1(b) &= 5 + 0.6 * 0 \\ U^1(b) &= 5 \\ U^1(a) &> U^1(b) \end{aligned}$$

Now in time=1, Martine will still choose A over B. Martine isn't impulsive, since she chooses A. She still gets a higher utility of A in time=1 then choice B.

b)

Being naïve means that John bases his choices inaccurate that his preference in the future is the same as his preferences today. To use the hyperbolic discounter model, we need to use the beta-delta function. It is close to the delta-function earlier but takes into account another parameter, β . The intuition is taking into account this parameter is that people are time inconsistent. People might change preferences as time passes. Beta-delta function is given as in this case:

$$U^0(u) = u_0 + \sum_{i=1}^2 \beta \delta^i u_i$$

Starting with looking at Johns utility in time t=0:

$$U^0(a) = 0 + 0.3 * 1 * 2 + 0.3 * 1^2 * 8$$

$$U^0(a) = 3$$

$$U^0(b) = 0 + 0.3 * 1 * 5 + 0.3 * 1^2 * 0$$

$$U^0(b) = 1,5$$

$$U^0(a) > U^0(b)$$

John will choose choice A in time=0, since it gives a higher utility.

Now time=1 has come. Now let's look at Johns utility from the perspective of t=1.

$$U^1(a) = 2 + 0.3 * 1^1 * 8$$

$$U^1(a) = 4,4$$

$$U^1(b) = 5 + 0.3 * 0 * 1$$

$$U^1(b) = 5$$

$$U^1(b) > U^1(a)$$

In time=1, John will choose choice B over A. Since $\delta = 1$, he doesn't discount future, but $\beta = 0.3$ discount his preference. Looking at choice in time=0 and time=1, John is impulsive but patience. He first chooses A in time=0. Choosing a comfortable retirement. But when time=1 comes, he changes to B. He gets a higher utility not saving money in time. Therefore, he will be poor during retirement. Being naïve he chooses not to save for retirement, even though he has a higher utility of choice A in time=0.

Note here is that since $\delta = 1$, you could say that John is an exponential discounter by swapping δ with β .

$$\beta = \delta = 0.3$$

Question 5

a)

Finding expected utility from the gambles:

$$\text{Expected utility function} = \text{Pr}(s) * \sqrt{x}$$

$$(A): u(A) = \frac{1}{4} * \sqrt{50} = 1.7677$$

$$(B): u(B) = \frac{1}{5} * \sqrt{100} = 2$$

$$(C): u(C) = \frac{1}{8} * \sqrt{150} = 1.53$$

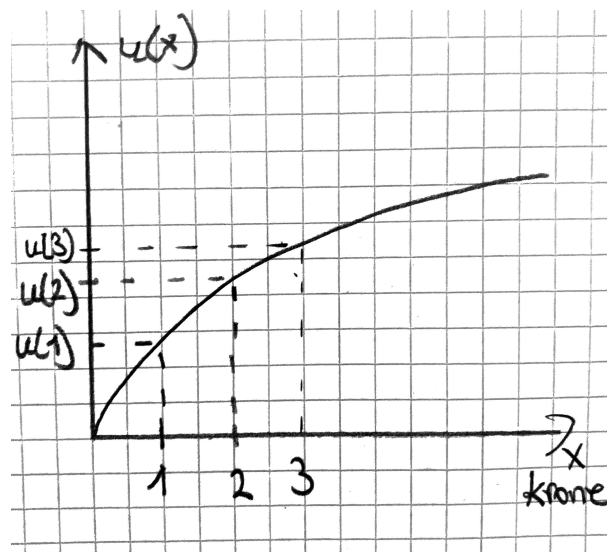
I would choose gamble B, since it gives me the highest utility of 2.

b)

Using utility function $u(x) = \sqrt{x}$ means the person is risk averse. This means that another kroner gives you higher utility, but the marginal utility of another krone is diminishing. Another krone doesn't give the same effect as the previous krone. We can write this mathematical as by taking the derivate of x .

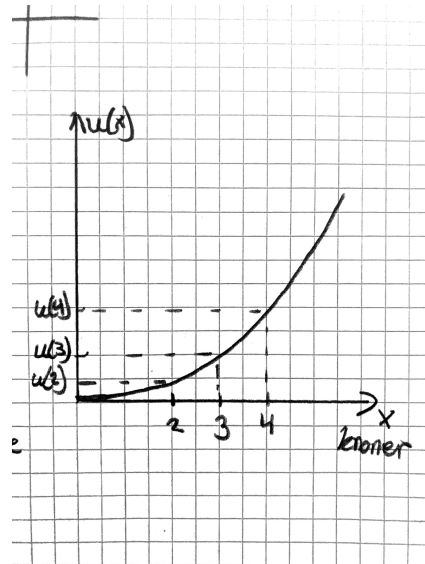
$$u'(x) > 0 \text{ and } u''(x) < 0$$

This can be drawn as.



A risk averse person will value another krone less in terms of utility than the previous krone.

Someone with the utility function $u(x) = x^2$ is a person that is risk prone. The person will value getting another kroner more than the previous krone. Its utility function is upward bending with these characteristics, $u'(x) > 0$ and $u''(x) > 0$. Draw like this:



A risk prone will take upon a gamble even if the expected value of the bet is zero. Using the example in task a to see what a risk prone person would choose.

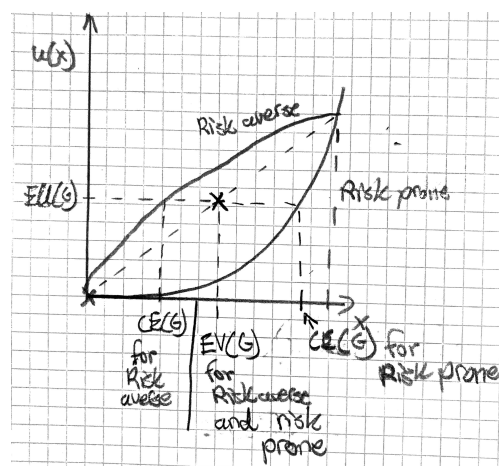
$$(A): u(A) = \frac{1}{4} * 50^2 = 625$$

$$(B): u(B) = \frac{1}{5} * 100^2 = 2000$$

$$(C): u(C) = \frac{1}{8} * 150^2 = 2812.5$$

If the person is risk prone, the person would choose gamble C. The person value winning more money.

You could also show this by computing the certainty equivalent of a gamble. Certainty equivalent of a gamble is what satisfies expected utility equal the utility of the certainty amount.



Question 6

a) Standard theory is that the proposer wants to maximize her dollar payoff, and that $u(x)=x$.

If this is the case then, the proposer should propose 1/6 equivalent of 10 NOK. Why?

Assuming both players have the same utility function, receiver is better off accepting any amount higher than 0 using backward induction. If she were to reject, she gets nothing.

Accepting gives a higher utility than rejecting the minimum amount proposer proposes. The proposer gets 54 NOK. This equilibrium is a Nash equilibrium. This assumes that the receiver doesn't have a strategy behind her choice.

b)

The proposer might choose a different amount depending on her social preference. This assumes that the proposer doesn't only care about her utility, but care about receivers' utility also. Previously we assumed the utility was $u(x)=x$, but the proposer might have a utility with multiple arguments. There are multiple preferences that the proposer might have.

Altruistic preference, *example* $u_p(x, y) = \frac{3}{5} * \sqrt{x} + \frac{2}{5} \sqrt{y}$:

Here the proposer is willing to offer some of its payoff to improve the receivers' payoff.

Or the proposer might be envious which gives this function: $u_p(x, y) = \sqrt{x} - \sqrt{y}$:

With this function, the proposer gets a better utility if the receiver gets a lower amount.

The proposer might have a Rawlsian preference, which means she wants to maximize the minimum utility of sharing the money. The function can be written as: $u_p(x, y) = \min(\sqrt{x}, \sqrt{y})$. Then the amount would be shared.

Another is utilitarian preference which is a variant of altruistic preference, $u_p(x, y) = \sqrt{x} + \sqrt{y}$. Here the proposer weights each amount each person gets the same.

Envious proposer prefers to get all the money, while a utilitarian and Rawlsian preference makes so that the proposer proposes equal amount between the two.

Another factor is that the proposer might think that the receiver looks at the proposers' intention. Receiver might have reciprocity where they reward the proposer's intention. Positive reciprocity means the receiver reward the proposer that have good intentions, and visa versus for negative reciprocity.