#1

a) $\int (3x^2+4x+3)\,dx = \underline{\underline{x^3+2x^2+3x+C}}$

b) $\int x e^x \,dx$

Bruk at $\int f g' \,dx = fg - \int f' g \,dx$ og

sett $x = f$ og $e^x = g'$:

$\int x e^x \,dx = x e^x - \int e^x \,dx = \underline{\underline{x e^x - e^x + C}}$

c) $\int \frac{x^2}{x+1}\,dx$

Sett $u = x+1$, $du = dx$

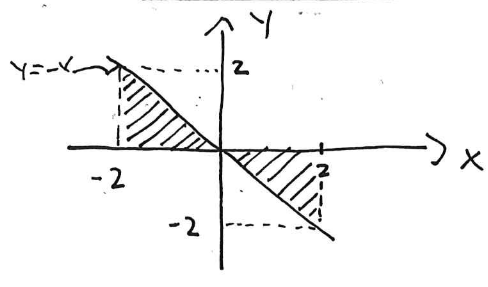
$$\int \frac{x^2}{x+1}\,dx = \int \frac{(u-1)^2}{u}\,du = \int \frac{u^2-2u+1}{u}\,du$$

$$= \int \left(u - 2 + \frac{1}{u}\right) du = \frac{1}{2}u^2 - 2u + \ln|u| + C$$

Sett inn for $u = x+1$:

$$\int \frac{x^2}{x+1}\,dx = \underline{\underline{\frac{1}{2}(x+1)^2 - 2(x+1) + \ln|x+1| + C}}$$

d)



$$\int_{-2}^{0} -x\,dx - \int_{0}^{2} -x\,dx = \int_{-2}^{0} -x\,dx + \int_{0}^{2} x\,dx$$

$$= \left[-\frac{1}{2}x^2 + C_1\right]_{-2}^{0} + \left[\frac{1}{2}x^2 + C_2\right]_{0}^{2} = -\frac{1}{2}\cdot 0^2 + \frac{1}{2}(-2)^2 + \frac{1}{2}\cdot 2^2 - \frac{1}{2}\cdot 0^2$$

$$= \frac{1}{2}\cdot 4 + \frac{1}{2}\cdot 4 = \underline{\underline{4}}$$

#2

$$
\begin{aligned}
2x_1 \qquad\quad + 3x_3 &= 4 \\
5x_1 + x_2 + 6x_3 &= 11 \\
9x_1 + x_2 + 12x_3 &= 20
\end{aligned}
$$

a)
$$
\left[\begin{array}{ccc|c} 2 & 0 & 3 & 4 \\ 5 & 1 & 6 & 11 \\ 9 & 1 & 12 & 20 \end{array}\right] {-7} \sim \left[\begin{array}{ccc|c} 2 & 0 & 3 & 4 \\ 5 & 1 & 6 & 11 \\ 4 & 0 & 6 & 9 \end{array}\right] {-2} \; {-2} \sim
$$

$$
\circledast \left[\begin{array}{ccc|c} 2 & 0 & 3 & 4 \\ 1 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{array}\right]
$$

Ser av siste rad at systemet ikke kan ha noen løsning.

b)
$$
\left[\begin{array}{ccc|c} 2 & 0 & 3 & 4 \\ 5 & 1 & 6 & 11 \\ 9 & 1 & 12 & 19 \end{array}\right] {-1} \sim \left[\begin{array}{ccc|c} 2 & 0 & 3 & 4 \\ 5 & 1 & 6 & 11 \\ 4 & 0 & 6 & 8 \end{array}\right] {-2} \; {-2} \sim
$$

$$
\left[\begin{array}{ccc|c} 2 & 0 & 3 & 4 \\ 1 & 1 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right] {-2} \left[\begin{array}{ccc|c} 0 & -2 & 3 & -2 \\ 1 & 1 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 0 & 1 & -3/2 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] {-1} \sim
$$

$$
\circledast\circledast \left[\begin{array}{ccc|c} 1 & 0 & 3/2 & 2 \\ 0 & 1 & -3/2 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]
$$

Sett $x_3 = s$:

$x_1 + 3/2\, s = 2 \Leftrightarrow x_1 = 2 - 3/2\, s$

$x_2 - 3/2\, s = 1 \Leftrightarrow x_2 = 1 + 3/2\, s$

$x_3 = s$

$$
\underline{\underline{X = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + s \begin{bmatrix} -3/2 \\ 3/2 \\ 1 \end{bmatrix}}}
$$

c) Fra $\circledast$ ser vi at vi har $r(A) = 2$, mens $r(A_b) = 3$. $r(A) < r(A_b)$ og systemet har ikke løsning. Fra $\circledast\circledast$ ser vi at $r(A) = r(A_b)$. Det betyr at vi har løsning på systemet. Fordi $r(A) = r(A_b) = 2 < n = 3$, har systemet $3 - 2 = 1$ frihetsgrad ($x_3 = s$).

3)

$$\dot{x} = 2x - 5y - 3$$
$$\dot{y} = 2x - 4y - 2$$

a) Likevekt krever at $\begin{bmatrix}\dot{x}\\\dot{y}\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$:

$$\underbrace{\begin{bmatrix}2 & -5\\2 & -4\end{bmatrix}}_{A}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}3\\2\end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\cdot\begin{bmatrix}-4 & 5\\-2 & 2\end{bmatrix} = \frac{1}{2}\begin{bmatrix}-4 & 5\\-2 & 2\end{bmatrix}$$

$\Rightarrow$ $\begin{bmatrix}x\\y\end{bmatrix} = \frac{1}{2}\begin{bmatrix}-4 & 5\\-2 & 2\end{bmatrix}\begin{bmatrix}3\\2\end{bmatrix} = \begin{bmatrix}-1\\-1\end{bmatrix}$

Likevektspunktet er $\underline{\underline{\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}-1\\-1\end{bmatrix}}}$.

b) I hht teorem 6.6.1, s244 i SHSS, har vi at hvis $tr(A) < 0$ og $|A| > 0$ er likevekten g.a.s.:

$tr(A) = 2 + (-4) = -2 < 0$ $|A| = 2(-4) - 2(-5) = 2 > 0$

$\Rightarrow$ punktet $\begin{pmatrix}-1\\-1\end{pmatrix}$ er g.a.s.

Alternativt, ved bruk av egenverdier:

$$A - \lambda I = \begin{bmatrix}2-\lambda & -5\\2 & -4-\lambda\end{bmatrix}$$

$$P(\lambda) = (2-\lambda)(-4-\lambda) - 2(-5) = \lambda^2 + 2\lambda + 2$$

$$\lambda_{1,2} = \frac{-2 \pm \sqrt{4 - 4(1)(2)}}{2} = \underbrace{-1}_{\text{realpart} < 0} \pm i$$

$\Rightarrow$ likevekten er stabil da den reelle delen av λ_1 og λ_2 er negativ.

#4

a) $\max_{x_1, x_2} U(x_1, x_2) = a \ln x_1 + (1-a) \ln x_2$

s.t. $\quad p_1 x_1 + p_2 x_2 = I$

$$\mathcal{L} = a \ln x_1 + (1-a) \ln x_2 - \lambda (p_1 x_1 + p_2 x_2 - I)$$

$$\mathcal{L}_{x_1} = \frac{a}{x_1} - \lambda p_1 = 0 \iff x_1 = \frac{a}{\lambda p_1}$$

$$\mathcal{L}_{x_2} = \frac{1-a}{x_2} - \lambda p_2 = 0 \iff x_2 = \frac{1-a}{\lambda p_2}$$

$$\mathcal{L}_{\lambda} = -p_1 x_1 - p_2 x_2 + I = 0$$

Sett inn for x_1 og x_2:

$$-p_1 \frac{a}{\lambda p_1} - p_2 \frac{1-a}{\lambda p_2} + I = 0 \iff \lambda = \frac{1}{I}$$

$$\hookrightarrow \underline{\underline{x_1 = \frac{a \cdot I}{p_1}}}, \text{ og } \underline{\underline{x_2 = \frac{(1-a) I}{p_2}}}$$

b)

$$\min_{x_1, x_2} \; p_1 x_1 + p_2 x_2$$

s.t

$$a \ln x_1 + (1-a)\ln x_2 = \bar{u}$$

$$\mathcal{L} = p_1 x_1 + p_2 x_2 - \lambda\left(a \ln x_1 + (1-a)\ln x_2 - \bar{u}\right)$$

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$$\mathcal{L}_{x_1} = p_1 - \lambda \frac{a}{x_1} = 0 \iff x_1 = \frac{\lambda a}{p_1}$$

$$\mathcal{L}_{x_2} = p_2 - \lambda \frac{(1-a)}{x_2} = 0 \iff x_2 = \frac{\lambda(1-a)}{p_2}$$

$$\mathcal{L}_{\lambda} = -a \ln x_1 - (1-a)\ln x_2 + \bar{u} = 0$$

Sett inn for x_1 og x_2:

$$-a \ln \frac{\lambda a}{p_1} - (1-a)\ln \frac{\lambda(1-a)}{p_2} + \bar{u} = 0 \iff$$

$$\ln\left(\left(\frac{a}{p_1}\right)^a \lambda^a \left(\frac{(1-a)}{p_2}\right)^{1-a} \lambda^{1-a}\right) - \bar{u} = 0 \iff$$

$$\ln\lambda + \ln\left(\left(\frac{a}{p_1}\right)^a \left(\frac{(1-a)}{p_2}\right)^{1-a}\right) = \bar{u} \iff$$

$$\lambda = \left(\frac{p_1}{a}\right)^a \left(\frac{p_2}{1-a}\right)^{1-a} e^u$$

$$\hookrightarrow x_1^c = \left(\frac{p_1}{a}\right)^{a-1} \left(\frac{p_2}{1-a}\right)^{1-a} e^u$$

$$x_2^c = \left(\frac{p_1}{a}\right)^{a} \left(\frac{p_2}{1-a}\right)^{-a} e^u$$

c) Bruker Marshalletterspørselsesfunksjonen og får

$$\frac{\partial x_1}{\partial p_1} = \underline{\underline{-\frac{aI}{p_1^2}}} \quad \text{(ettersp. går ned)}$$

d) Slutsky-likningen sier at

$$\frac{\partial x_j(\bar{p},I)}{\partial p_i} = \frac{\partial x_j^c(\bar{p},u^*)}{\partial p_i} - \frac{\partial x_j(\bar{p},I)}{\partial I}\frac{\partial E(\bar{p},u^*)}{\partial p_i}.$$

Her er $i = j = 1$.

Første del av H.S. er substitusjonseffekten.

1 Sett inn for x_1 og x_2 og finn

$$u^* = v(p_1,p_2,I) = a\ln\frac{aI}{p_1} + (1-a)\ln\frac{(1-a)I}{p_2}$$

$$= \ln\left(\left(\frac{aI}{p_1}\right)^a\left(\frac{(1-a)I}{p_2}\right)^{1-a}\right)$$

$$\frac{\partial x_1^c}{\partial p_1} = (a-1)p_1^{a-2}\left(\frac{a p_2}{1-a}\right)^{1-a} e^u.$$

Innsatt for u^* gir:

$$\frac{\partial x_1^c}{\partial p_1} = (a-1)p_1^{a-2}\left(\frac{a p_2}{1-a}\right)^{1-a}\overbrace{\left(\frac{aI}{p_1}\right)^a\left(\frac{(1-a)I}{p_2}\right)^{1-a}}^{e^{u^*}}$$

$$= (a-1)a\frac{1}{p_1^2}I = \underline{\underline{(a^2-a)\frac{I}{p_1^2}}} \quad (<0)$$

2

$$\frac{\partial x_1}{\partial I} = \frac{a}{p_1}$$

Vi vet fra omhyllingsteoremet at $\frac{\partial E}{\partial p_i} = x_i$.

Da blir inntektseffekten

$$\frac{\partial x_1}{\partial I} \cdot \frac{\partial E}{\partial p_1} = \frac{a}{p_1} \cdot \frac{\overbrace{aI}^{x_1}}{p_1} = \left(\frac{a}{p_1}\right)^2 I$$

e) Slutskylikningen gir at

$$\frac{\partial x_i}{\partial p_i} = \frac{\partial x_i^c}{\partial p_i} - \frac{\partial x_i}{\partial I} \cdot \frac{\partial E}{\partial p_i}$$

$$= (a^2 - a)\frac{I}{p_1^2} - \left(\frac{a}{p_1}\right)^2 I = -\frac{aI}{p_1^2}$$, jfr sp. c.